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Class Philosophy
Sub-class Logic
Language English
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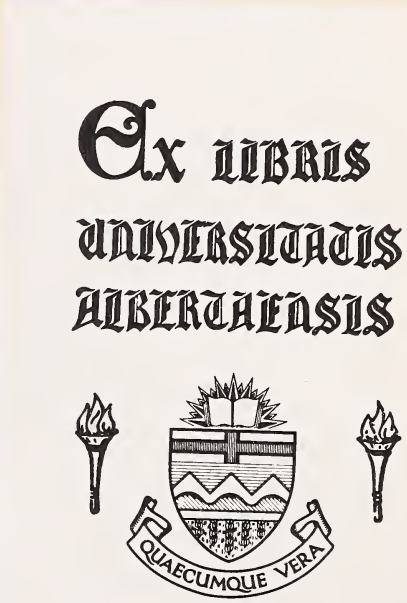

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Proof - one form of premise or argument admitted as true, major term

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THE SCIENCE OF THOUGHT.

LOGIC

A SYSTEMATIC EXPOSITION OF THE NATURE, METHOD, AND RELATIONS OF HUMAN THOUGHT

BY

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IVISON, BLAKEMAN & COMPANY,
PUBLISHERS,
NEW YORK AND CHICAGO.

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PREFACE.

THE leading characteristics of the author's presentation of logical science appearing in this treatise are the following :—

1. The science is squarely based on a familiar fact of experience, being an instance of thought containing the essential nature of thought.

2. From the essential nature of thought thus apprehended the fundamental laws of thought are directly evolved.

3. The Judgment is recognized as the one mature and perfect form of thought or discursive knowledge.

4. The Concept is shown as an organic member of the Judgment, and as having its genesis with it and in it, so that the judgment is not before the concept in origin nor the concept before the judgment, but as any living body and member they necessarily come into being together.

5. The Reasoning is a derivative of the

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Judgment and is not therefore to be co-ordinated either with it or with the concept as a third form of thought.

6. The Inductive movement of Thought is co-ordinated with the Deductive movement in equal rank and equal claims to a full elaboration of its nature and forms and method of procedure. In fact, in view of the hitherto defective treatment of this process of thought which in these recent times is the justly vaunted instrument and power in the advancement of science, what would otherwise be regarded as a disproportionate consideration is here given to it with exemplifications in the several wholes in which the movement may be made.

7. The Categories of pure thought are exhibited in their source and genesis, with their rank and authority as first or highest principles of truth or knowledge, yet not innate nor apriori to thought, but only as the intrinsic attributes of thought discernible in actual experience.

8. Thought is exhibited as an organic product and consequently as more or less determined by its outer organic relation-

ships, especially to the real universe as the correlative object to thought.

9. The ideal of a perfect science, of a science which should be an exemplification of a true and worthy scientific treatment of whatever subject, grasping firmly its matter, and developing it in exactest method through all its generic relationships in their relative order and connection to its proposed end or aim, has been before the author's mind throughout the work. Deeply sensible of the degree to which he has fallen below his standard he flatters himself that even in its imperfection the work may help to suggest what a worthy science of thought ought to be both for its own sake and also as a model for every particular science.

While a commanding object has been the preparation of a text-book for learners which should exhibit the science of human thought in a full, yet compact and methodical form, it has also been a prominent object to lead the student into an abiding and ever strengthening habit of right thinking. To be strong intellectually, it is necessary not only to think much, but also to

think habitually in the lines of legitimate thought and with the constant inspiration coming from the intelligent assurance that the thinking is right thinking and conducts to truth. The study of logic as a normative science should serve both as an incentive and a help to such habitual thinking. Copious exercises in the different movements of thought are presented in the appendix.

New Haven, 1886.

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INTRODUCTION.

DEFINITION OF LOGIC—METHOD—HISTORY—UTILITY.

§ 1. LOGIC IS THE SCIENCE OF THOUGHT.—The term comes from the Greek, λόγος—a word which, in accordance with the analogies of language, while more properly perhaps signifying the uttered or spoken thought—the thought as embodied in vocal sound—was freely used also to denote the mere thought itself. So Aristotle distinguished between the outer and the inner λόγος. The science, thus, is denominated from its subject-matter—thought itself. Logic, accordingly, the science of thought, is to be conceived not as setting forth some peculiar way of thinking, but as the way of all legitimate thinking, to whatever subject thinking may be applied.

Logic belongs to the class of sciences not inaptly termed by Sir William Hamilton *Nomological*, or still more significantly, *Normative*. It is a science of norms, that is, of laws deduced from the essential nature of its subject-matter and also of forms as determined by these laws. Its more characteristic function is to set forth rules and

typical forms. To restrict it to a consideration of the mere laws of thought, to define it as "the science of the laws of thought," is an inadmissible narrowing of its province. It is more truly the science of thought considered not only in its regulative principles, but also in its processes, its methods, its organic relations—of thought in the full broad significancy of the term. It constitutes with Æsthetics and Ethics that special class of normative sciences which concern themselves respectively with the three grand departments of mental phenomena. These departments are denominated subjectively, or from the mind itself, as the Intelligence; the Sensibility and Imagination; and the Will. They are also denominated objectively, or from the objects which these mental functions respect, as the True; the Beautiful; and the Good. It is the special characteristic of these sciences as Normative that they more immediately regard the products of an active nature, and not the active nature itself or the functional activity, as the Intelligence, the Imagination, the Will, nor yet the object which these functional activities respectively address, as the True, the Beautiful, the Good. Logic regards thus thought as a completed act, not the action of thinking nor the object which is thought. The legitimate forms of this product, of thoughts as acts, are proper *norms*, which may be viewed either as to the mode of their formation in the aspect of laws or rules

or as to the intrinsic character of the products themselves, in the aspect of types or typical forms. Logic, thus, is a proper normative science, which as dealing with products of an activity prescribes the governing rules and the legitimate forms of thought.

As a normative science characteristically having to do with rules and forms, it approaches to the character of an art, as art is distinguished from science. But a proper art regards the activity itself—prescribing that, regulating that, training that, rather than the product. Logic is preferably to be viewed as a pure science which satisfies itself with a full exposition of the nature of thought through its governing principles and legitimate forms in their essential character and methods, and only in a subordinate way regards the training, regulating, and using of the faculty of thinking. It sets forth the laws of thought but only as they are those essential attributes of thought which express its true nature. It may prescribe rules for particular forms of thought, and furnish exercises in the shaping of them; but its governing principle will be teaching rather than training. Its end is knowledge; while the end of an art is skill. As a normative science it however approaches so near a proper art, that in fact logicians have been divided in their opinions whether to class it as a science or an art, and their actual expositions hardly indicate in which of these two classes they

intended to place it. For real uses in study they are hardly to be distinguished; and the student may regard them at will without danger of serious error or harm under either denomination. A normative science and a proper art must alike set forth rules, determine forms, and exemplify in suitable exercises.

Logic, further, is an *exact* science. It is its high prerogative, if the paradox may be allowed, to be its own creator. It has accordingly been styled the science of all sciences—*scientia scientiarum*. It is the supreme arbiter throughout the realm of knowledge. From it there is no appeal. Its decisions are conclusive. It exacts implicit and full obedience to all its requirements. It is just because thought is the creator of the science of thought and so dominates with absolute sway throughout the domain of knowledge, that its legitimate procedures are stamped with the mark of necessity. The rules and forms of logic, and they alone, give *necessary* truth, because to question its work is to reject the sway of thought itself and cast it down from its throne. For legitimate thought to question itself, is suicidal. Thought is no more thought when it thus rebels. How can I think when I think what is not true thought—when my thinking is not thinking? Logic is thus an exact science, like pure mathematics, which is so only because, as will be seen, it deals with a product of pure thought only and deals with it only in conformity with the inexor-

able rules of thought. As such a science, logic carefully excludes all that is foreign to thought—all that is not thought. It thus exactly defines its subject-matter, separating it from everything else, while retaining all its proper essence. It proceeds in its unfoldings of this its subject-matter in exact method; that is, in strict conformity to the principles of thought. As will be seen, all that can be thought must be recognized as being in the relationships of whole and part. The method in logical science will accordingly observe the exact relationships of parts to one another and to the whole in their orderly and congruous juxtaposition and interdependence, as in a perfect organism. It will in this way proceed to its proposed end, the exhibition of the nature and characteristics of perfect knowledge, unfolded in its rules and its forms, in all its divers generic modifications, giving at once the marks and tests of true knowledge. Logic is an exact science, thus, as knowing no rule but its own and as defining its own province, its own method, its own end, allowing no external dictation or interference. It admits no assumptions, no truths, no axioms, no principles before itself. It begins from simple thought, and all it asks is the *datum* of a single instance of thinking. Out of this germ it evolves the whole science of thought, borrowing nothing, needing nothing, demanding nothing—unless it be simply its needful body of language in which it may grow and

properly express itself, for like the human soul itself, human thought demands a body in which to live; while, moreover, it claims itself to be the parent and former of this its body and instrument. In this pure domain of pure thought in which no foreign interest can find a lawful settlement, logical science may rightfully claim to be sole sovereign and so be exacting everywhere, admitting nothing but thought, genuine thought, pure thought, in its essence, in every specific form, and in respect to its single end, thought in its exactest import everywhere. It starts from thought, moves along the straight line of thought, and leads to its goal of thought. Its germ, its growth, its fruitage, are only exactest thought. Of no other science, except, as intimated, mathematics, can this be said. These are the two exact sciences.

§ 2. Thought, as thus the subject-matter of logical science, may be defined to be *the product of the intelligence in its mature or completed stage.*

The parts of this definition require separate consideration.

First, thought is here to be regarded as a product. The term *thought* is familiarly and properly used to denote the faculty of thinking; as when we say, 'the thought of man cannot comprehend the nature of God.' It denotes, also, the object which the action of this faculty may respect; as, 'the thought was overwhelming,' that is, the object as thought,—the supposed calamity

—was overwhelming. Still again, it denotes the exercise of this faculty; as, 'my protracted thought wearied me.' Once more, it denotes the product of this action upon the object; as, 'the thought was correct.' It is in this last sense in which the term is to be taken as denoting the subject-matter of logic. It is only in an indirect and incidental way that logic regards the faculty of thinking itself, or its specific exertion, or the object,—the true, which the faculty addresses.

Secondly, human thought is the product of that faculty or function in man, which is known as the Intelligence or Intellect; the Cognitive Faculty; the Understanding.

Thirdly, by thought as the subject-matter of logical science, is to be understood only the product of the Intelligence in its mature and completed stage. There have been recognized two distinct stages of knowledge. The inchoative, germinant, preliminary stage, has been designated as *presentative* knowledge, or *simple apprehension*. In order to be fully known, the object must be presented to the mind; it must be apprehended by it in some way. And this state of the human mind has been recognized, perhaps somewhat loosely, as a state of knowledge—a cognition. It is of two kinds distinguished as to its object, whether lying without the mind or within the mind. The former is called a *Perception*—perceptive or sensible cognition or knowledge. The latter is called an *Intuition*—an intuitive cog-

nition or knowledge. The object thus presented in simple apprehension, whether external as in a perception or internal as in an intuition, is, in reference to proper thought which is the second, the mature or completed stage of knowledge, technically called the *datum* to the thought.

But the human mind does not rest with this merely inchoative, incomplete knowledge. By a strong natural instinct it presses on to a second stage—to proper thought, in which it recognizes in this *datum*, this presentative cognition or simple apprehension of an object, some attribute, which it thinks as belonging to the object. As if an *orange* be perceived by the mind, from the apprehension or perception of the object the mind passes to recognize it as *yellow*, as *round*, and the like. It reaches a completed stage of cognition, when it thinks that *the orange is yellow, is round*, or the like. The cognition now is a cognition of an object as having an attribute or attributes. This is proper thought—KNOWLEDGE BY AN ATTRIBUTE. It is sometimes called, to distinguish it from a presentative knowledge, a *representative* knowledge. It is also called a *reflective* knowledge, because the mind bends itself back to regard the object given in the apprehension—the *datum* of the perceptive or intuitive knowledge—as already in the mind, already apprehended. It has also been designated as the product of the Discursive Intelligence, and hence called a *discursive knowledge*. Still further, while

a presentative knowledge is denominated an immediate knowledge, this reflective, discursive knowledge, or thought, is styled a mediate knowledge, since the knowledge involves the introduction into the mental experience of an attribute by which in fact the completed knowledge or thought is mediated. In thought, accordingly, an attribute ever appears over against the object, or, as it has now become in this distinguishing act of reflection, the subject to which the attribute belongs. It is carefully to be remarked that the attribute is in a true, the truest and highest sense, the same as the subject—the yellowness is not one distinct thing, and the orange another. They are one and the same. The orange *is* yellow. It is the same in one respect; it is one and the same in part, as, for instance, in respect of its color.

Knowledge is thus made up of the two stages or constituents—presentation and attribution; and thought as the subject-matter of logical science is the completed stage or proper attributive knowledge.

It will need however to be carefully noticed that the thought which constitutes the subject-matter of logic is *pure* thought; that is, thought abstracted from the object of thought. When we think, we think of something. The thinking activity respects some object. A subjective element meets and fuses itself, so to speak, with an object; thought unites itself to thing. The

characteristics of these respective elements widely differ. They can be separated for our convenience in our reflective studies upon them. In fact, as will be shown in the proper place, there are in every instance of a thought three distinct elements, each having its own peculiar properties, which it becomes necessary in nice investigation carefully to discriminate. There is in every such case, first, the subjective element or that of thought itself with its characteristic attributes, secondly, the objective element, that which the thought respects as its object—that of which we think, which has its peculiar properties, and, thirdly, the resultant from the meeting of the subject element with its object—the thought thing, which has certain attributes that are neither those pertaining to the thinking nature nor to the object which it regards, but that characterize the product of the thinking activity as exerted on its object. It is this product, not the thinking action, not the object on which the thinking is exerted, but the product or result with which logic as the science of pure thought, 'of thought as thought,' has to do. Still farther, its view is directed on the thought-side of this product or result of thinking upon an object, excluding the object-side. It is yet true that the thought itself is necessarily modified by the special nature of its object. In order to the very existence of thought there must be supposed a correspondence between thought and its object of

a truly organic character. This relative nature of thought which is discerned in its necessary relation to its object must not be wholly lost sight of in the science of thought. It will disclose itself in the course of a development of the science. Only, nevertheless, so far as necessary to the very being of thought will this relative or extrinsic attribute, except at least incidentally and in subserviency, come into distinct consideration. It cannot be regarded as pushing unwarrantably the import of the term *pure* to bring within its scope as applied to thought the consideration of the necessary extrinsic attributes as well as those which are strictly essential and intrinsic.

The Method of Logical Science.

§ 3. The general character and direction of the method proper to Logic are indicated in what has been already said of its special nature as a science and of its subject-matter. The science of thought should first take as its fundamental *datum* some special instance of thought which shall conveniently reveal its essential character. This essential character of thought must of course appear in all thought, in every instance of thought. This essence of thought, too, must include in itself all the necessary constituents of pure thought, all its characteristic attributes. These attributes constitute the essential laws of thought

inasmuch as in order to genuine thought these attributes must appear in it and determine it. The fundamental laws of thought can be in fact nothing else than the different phases or aspects of its essential nature. Essential attributes are the true laws in all things. It is little else than a truism to say that a thing must possess its essential attributes, which accordingly give it its fundamental laws. Gravitation is a law of matter simply and only because gravity is an essential attribute of matter.

The fundamental laws of thought being ascertained as diversified expressions of the essential attributes of thought, its governing or generic forms are to be determined under the guidance and control of these fundamental laws. Such forms as the nature of thought under these laws shall disclose will need to be expounded, both in relation to their origin or genesis, that is, their legitimate production in a true process of thought, and also in relation to one another as organic parts constituting together the entire body of thought-processes. The consideration of the fundamental laws and generic forms of thought, in which their proper character and relations will appear as THE ELEMENTS OF THOUGHT, will accordingly constitute the First Part of Logical Science.

But thought pertains to a rational nature. All true thinking is rational, implying an end; and the one proper end of all thinking as a rational

procedure is knowledge or truth. Logical Science is required accordingly not only to expound the essential nature and legitimate forms of thought, but also to set forth the proper use of these forms in order to accomplish the end of all rational thought which is truth. The consideration of the proper use of the elements of thought in reference to this one end of all rational thinking—the attainment of truth—will thus constitute the Second Part of Logical Science under the denomination of THE METHODS OF THOUGHT.

Once more, thought is the product of an active nature which necessarily requires an object upon which it shall exert itself. The great universe of the true, of objects that can be thought or known, constitutes this comprehensive object for the activity of the thinking nature. There can be no thought without such object. This object of thought determines thought as to its character and method. The true as object and thought as subject are in vital organic relationship to each other; and neither can be rightly understood irrespectively of the other. The consideration of *the true* as the comprehensive object of thought as a whole and in its special departments marking out the particular branches of knowledge or science, will accordingly constitute the Third Part of Logical Science under the denomination of THE RELATIONS OF THOUGHT.

History of Logical Science.

§ 4. Logic does not appear to have assumed any proper scientific form until the time of Aristotle. The nature of knowledge, the legitimate subjects of knowledge to man, the forms of knowledge, the processes, tests, and uses of knowledge must, however, have engaged the consideration of profound thinkers even in the infancy of philosophical thought. In the Hindoo Philosophy, we are told in Prof. Max Müller's account of Indian Logic, although there seems to have been no formal treatment of logic worthy of the name of Science, there yet are to be found hints and suggestions of logical interest. The reasoning or argument is recognized as consisting of three real members: the proposition, the reason, and the conclusion. This number of constituent propositions is increased to five if the assertion which afterwards appears as the conclusion is first named and is followed by the reason or ground, and then the full three-membered form of the regular syllogism is added. Thus, (1) *A is C* for (2) *B is C;* but if (3) *B is C;* and (4) *A is B;* then (5) *A is C.* An enumeration of thinkables is given also. Six, or, with the negative, seven, are enumerated: (1) Substance; (2) Quality; (3) Action; (4) Genus; (5) Individuality; (6) Concretion; (7) Non-existence. In this enumeration was to be found everything which could be known. Substance comprehended the five elements: earth,

water, light, air, and ether; also time and space; and self and soul. These are all things. The Second and Third in this list of the thinkables—Quality and Action—are evidently comprehensive of what may be thought of a substance in its intrinsic attributes;—they are predicables. Genus and Individuality are the obvious terminal parts of logical division. Concretion seems to have been understood as including that which may be thought merely in relation and so to have comprehended the relative or extrinsic attributes of a substance. Under Non-existence were comprehended the negative forms of knowledge. We discover even in this cursory glance what as germs may have been developed under Grecian thought into leading branches in Grecian logic.

§ 5. Aristotle (B.C. 384–322) is justly reckoned the father of logical science. As among the Hindoo philosophers, so also among the Grecian sages, much had been done in this field of knowledge; and Aristotle made good use of these labors of his predecessors. But the writings of these earlier thinkers have been ill-preserved, and their speculations seem to have been partial and fragmentary. To Archytas of Tarentum, who lived a half century before Aristotle, is attributed an enumeration of the ten thinkables which appear in what have been called the ten categories. Both Hindoo and Greek enumerations include not merely proper categories or predicables, but subjects of thought as well. This is obvious from

mere inspection. The first particular named in each enumeration is *substance* or *essence*, and the Hindoo specification under this gives, as we have seen, the five physical elements, also *time* and *space*, and *self* and *soul*, all of which are concretes. Aristotle distinguishes two classes of *substances* or *essences*, first and second, and says: "The chief substance—οὐσια—and that first and especially meant, is neither spoken of any subject nor is in any subject, as *a certain man, a certain horse.*" In fact, both enumerations are expressly stated to be lists of words—utterances of thought in speech regarded apart from any assertion. Of these Aristotle says: "Some are spoken in combination, others without combination; those in combination are such as *man runs; man conquers.* Those without combination are such as *man, horse; runs, conquers.*" These early studies into the nature and forms of thought started evidently from the thought as uttered in speech—from words. And words, of course, included both subjects, which were not carefully distinguished from concretes, and predicates. Aristotle, in his treatise entitled *Topics*, gives a list of what he expressly calls *categories*—predicaments or predicables. It embraces ten in number. The first is called in the list, not substance or essence, but, *the what is*, what the thing spoken of is; which, he adds, signifies substance or essence—οὐσιαν.

It has been claimed, indeed, that Zeno, the Eleatic philosopher, who flourished nearly a centu-

ry before Aristotle, was "the inventor of logic." But this claim seems to be entirely unfounded. Zeno wrote a treatise on *eristics*—the art of disputation—and is reputed to have first introduced the use of the dialogue in philosophical discussion. On these grounds he may with some show of reason be credited with the introduction of the *dialectic* art into Grecian thought. According to Aristotle, dialectics had to do only with probable questions, and is distinguished by him from syllogistics, or the art of demonstrative reasoning.

The treatises on logical subjects by Aristotle, so far as preserved to us, are five in number. They have been by his followers collectively denominated the Organon of Aristotle. They are not put together so as to form a system. They are rather detached treatises on distinct logical topics. The first, in the order in which the later editors have arranged them, is styled The Categories. Aristotle in this treatise discusses the nature and uses of words in relation to what they signify. The second treatise is on Interpretation, in which he treats of the nature and forms of propositions—of the noun, the verb, of the sentence and its kinds. The third treatise is The Topics, in eight books. Its design is stated to be to discover a method by which we shall be able to syllogize on any proposed question from probables. "It is useful," he says, "for three things: for practice, for conversations, for philosophical sciences." The fourth treatise is The Sophistical

Elenchi. It presents the modes of refutation in sophistical reasonings. The fifth treatise, in two books, is generally known as the Prior Analytics. Aristotle himself says it treats of Demonstration, and its object is Demonstrative or Apodictic Science. It is an elaborate work on the syllogism, its nature and forms. The sixth treatise is styled the Posterior Analytics. It is in two books, and treats of knowledge or truth in acquisition and communication.

Aristotle lived during the ripest days of the so-called age of the Sophists, himself contributing no little to the movement already begun by Socrates and Plato that resulted in their utter overthrow. What was at first the commendable pursuit of wisdom and the spread of knowledge had degenerated into empty charlatanry, in which men strove by trickeries in the use of legitimate forms of thought and of verbal expression, for purposes of gain or renown, to amuse, to startle, and to bewilder. Not conviction but marvel was the end, and not reason but illusion the means. Such was the spirit of the age. Aristotle seems, from what he tells us, to have looked upon this phenomenon of his age as he did on phenomena in the natural world. He spent much time in searching into these juggleries of wit and word to discover the secret of their power. For power there was in them; the best and strongest convictions of men were swayed by them. It was the power of reasoning—ratiocination. The human mind is

penetrated to its lowest depths with the feeling that there is truth, and that all truth is connected, one part with another, and that men are safe in following the lead of any particular truth to others associated with it. The expert sophist professed the art of proving and alike of disproving any proposition from any assumption. It was but knack with him, not genuine art which is founded in truth and fashioned out in truth. Aristotle's keen analytic eye penetrated beneath the surface of this charlatanry or play with word and wit and detected the secret of its power. His great discovery in logic was the discovery of the syllogism—the true art of reasoning—and his chief labor in the science was the elaboration of the syllogism as the last and highest and most comprehensive achievement of human thought, in its fundamental nature and legitimate forms, and as an instrument or means of attaining and imparting truth. The syllogism, he taught, was the only legitimate mode of deriving one truth from another with positive certitude; and syllogistic conclusions were of the nature of necessary truth, truth that it was impossible in reason to deny, if at least the basis of the reasoning were lawfully assumed. He defines syllogism at the start to be "discourse"—λόγος—using the term in its comprehensive import which embraced both inner and outer thought, or thought itself and thought expressed in words,—"discourse in which, certain things being posited, something else of necessity

comes to be by reason of what is posited." A perfect syllogism is one that requires nothing else besides what is assumed in order to the appearance of the necessary consequence. He everywhere carefully distinguishes the true syllogism—the syllogism which leads to necessary truth, which seeks true knowledge or philosophy, which is apodictic or demonstrative and not merely probable or but matter of opinion—from the dialectic which lies wholly in the field of merely probable matter. He defines its elements—its three propositions and its three terms and their relation to one another—as that the last should be in the whole of the middle and the middle in the whole of the first, or not; its three figures, since the middle term can be predicate of one term and be subject of the other, or be predicate of both the others, or subject of them—no other figure being supposable in a perfect syllogism, the fourth figure of the later logicians not being such. He lays down copious rules with exemplifications for the use of the syllogism, and shows that all reasoning in whatever subject-matter or for whatever purpose where truth is the aim is reducible to one or the other of the three figures named, pointing out how this reduction is legitimately to be made. He extends the principle of the syllogism to include the inductive as well as the deductive movement of mediated thought. In a way somewhat obscure to the modern mind because trained by long continued teaching to

construe all mediate reasonings only in extensive quantity, he exemplifies the application of this general principle to the inductive syllogism in Book Second and chapter twenty-fifth of the Prior Analytics, and indicates precisely how the movement is modified here. Aristotle in unfolding his doctrine of the reasoning and in his exemplifications, follows such a general or comprehensive method of treatment as to supersede the usually received distinctions of quantity into intensive and extensive, and moreover expresses the relation of predicate to subject in a manner rather foreign to our mode of thinking it. His treatises are therefore somewhat obscure. They are however characterized by the exactness of mathematical science.

Logic thus received from Aristotle its first shaping into a properly scientific form. It was a remarkably full and rich development for the beginnings of an abstruse science. So complete was his work that the centuries of thought which followed contributed little to its improvement. The syllogism has continued to form the body of the science of thought even to the present day, and with slight changes the Aristotelian mode of treatment has remained substantially the same, as to the divisions, the legitimate forms, and the uses of this movement of thought. But the work of Aristotle in this field was partial and fragmentary. It confined itself to a single form of thought. It was rather a scientific treatment of

dialectics—of disputatious thought, as a phenomenon of the age—than of thought generally in its essential nature and its organic parts and relations. It possessed however the vitality of truth—of truth in real practical relationship to the interests of human life,—and so has through the ages maintained its essential identity of form as well as its authoritative character in all departments of scientific study. In some such way, although in a fuller sense, Aristotle is to be styled the father of logical science as Euclid of the succeeding century has been styled the father of mathematical science. Geometry is to the whole field of this latter science what the syllogism is to the whole field of logical science or the science of thought.

To Theophrastus, who succeeded Aristotle in the presidency of the Lyceum, and Eudemus, another disciple of Aristotle, Boethius attributes the addition of five moods to the original four enumerated in the first of the syllogistic figures. In the time of Galen [A.D. 130–201], these five added moods had been transferred to a new figure, making the fourth.

§ 6. Commentators, editors, translators, compilers, all along down to the present time, have labored in the line of this originator of the science, with however but inconsiderable contributions to its enlargement and perfection. In the third century of our era, Porphyry [A.D. 233–304] wrote an "Introduction" to Aristotle's Categories, treat-

ing of the five denominations of Genus, Difference, Species, Property, and Accident, professedly repeating what had been taught from of old in regard to these comprehensive classes of predicables, as to their respective natures and uses.

Two centuries after, Boethius [A.D. 470–525] translated and expounded the Organon of Aristotle. He was for centuries held in great reverence in the schools of learning. Logic was now formally enumerated as one of the seven liberal arts, forming with Grammar and Rhetoric the so-called *Trivium*, the *Quadrivium* consisting of Arithmetic, Geometry, Astronomy, and Music.

From the eleventh century onward during the scholastic age the Aristotelian logic was cultivated with a most reverent diligence. Of the Arabian philosophers Averrhoes [A.D. 1149–1198 or 1206] translated and expounded the writings of Aristotle; while with the Scholastics the Stagirite was esteemed as "the philosopher" and the last authority in metaphysics and logic. It was in the thirteenth century that the famous logical verses —"Barbara, etc.," (§ 86)—appeared. They were the production of Petrus Hispanus, who was a native of Lisbon, became Pope of Rome, and died A.D. 1277.

§ 7. In the more recent times Logic, which had somewhat lost its hold on the leaders in science and philosophy, has risen to fresh life. In Germany since the days of Kant it has received the most diligent study. With the most of its culti-

vators, the substance of the Aristotelian doctrine has been retained. The Hegelian School however has given it entirely a new character by its identification of the thinking subject with the thought object. In Great Britain, since the publication of the little work by Archbishop Whately, logic has received a large attention. By far the most eminent of the British logicians is to be ranked Sir William Hamilton, whose lectures on Logic appeared in 1860 as a posthumous publication. The great contribution to logical science made by Hamilton consisted pre-eminently in his demolition of the whole formulastic structure as it appears in the *Barbara* verses, and the development of the logical doctrine of logical quantity in its respective forms of intension or comprehension and extension. But in the history of particular logical forms and teachings, and in the detection of errors or defects in previous systems, and especially in the shrewd penetrative glances which he threw into the more secret relationships of complex thought, his works place him at the head of the logicians of the present century. The works, as they are left to us, are however in a lamentably incomplete condition if regarded as a thorough-going system of logical doctrine.

A prominent characteristic of recent logic is the symbolistic treatment of the forms and processes of pure thought. In Great Britain, Germany, and Belgium, as also in the United States,

such symbolistic or 'algorithmic' systems have appeared. The endeavor in this is to represent the relations in thought and the allowable changes in them by letters as in Algebraic Science, with the use also of certain signs or characters symbolizing relations in quantity, as those of equality, addition, etc. They are ingenious and curious; but their practical utility is questionable. The use of them is extremely limited as compared with the use of the algebraic symbols. The translation of the thought to and from the symbol is slow and awkward. Moreover a mistake in regard to a single principle of thought, as it appears for example in some or all of these systems in the confusion of pure contradictory opposition in the copula with contrary opposition in the terms of a judgment, sheds doubt on every conclusion of a prolonged process of thought carried on by means of these symbols.

Utility of Logical Science.

§ 8. The utility of logical science may be estimated from the intrinsic excellence and interest of the science itself considered both in respect of its subject-matter and of its form, and also from its value as a mental discipline and an instrument of knowledge.

First, as a science in itself and in respect to its subject-matter—human thought—logic presents to every liberal and aspiring mind imperative in-

ducements to its study and cultivation. The mechanism of thought, its parts or elements, its springs, its movements, its limits, its products, its guides and principles,—no subject certainly possesses a prior claim to the consideration of thought itself than this its own nature and province and mechanism. All science is the product of thought; logical science is thus the science of science itself. It is accordingly the basic, fundamental science—the science of all knowledge. The first truth we gain is given us by thought, or more exactly is given us in thought. Observation —perception of external things and intuition of mental phenomena—gives us only what at best can worthily be called incomplete knowledge—elemental, rudimentary, embryonic knowledge. We attain to a true knowledge of anything only in reflection—in thought. Observation may accumulate its stores in immeasurable heaps; may sweep over the whole expanse of the real world and pierce to the remotest atom and take notice of its least palpable motion; but it is all comparatively worthless. Perhaps, indeed, it would be absolutely so, but for the anticipations of thought which even the simplest perception bears in itself, even as the bud prefigures in its inmost rudiments the leaf that is to be. The value of observation is to be found in the fact that it is the rudimentary, inchoative condition of thought. The starry heavens unfold themselves alike to all observing eyes. The facts of Astronomy

thus observed, even although somewhat developed and shaped by the meager thought which the human mind carries by an organic necessity in all its movements, are as nothing compared with the grand fabric of science which a few thinkers of the race have constructed. The plow or the spade upturns a fossil which the laborer observes, but it is to him a meaningless thing, conveys to him no knowledge or truth. The geologist reads in it a voluminous history of life, habits, conditions, laws, belonging to ages before the flood. In short, human intelligence would be little elevated and expanded above and beyond that of the brute but for the faculty of thought.

§ 9. It is to be remembered in our estimation of the value of the science of thought from this point of view—its subject-matter—that thought is a reality. Logic has to do with a true entity, not a mere abstraction. In this particular it differs from mathematical science which deals only with an abstract, a mere attribute of thought. Logic founds itself upon a fact in actual experience, not on an assumption, not on an axiom, not on a definition. It is a real, an actual, a concrete product of living power in positive exertion, which is the one germ of the science, which furnishes the single source of its entire development except as the diversity of its objects may modify and nourish out its forms and branches, just as the nucleus in the seed is the

source of life to all that the oak may ever become in its richest ramifications. The entire essence of thought is of course to be found in its simplest product. A world of interest is thus seen to surround the development of the science from the fact that out of the extremest simplicity is unfolded an immeasurable richness of form; inasmuch as the entire essence and nature and laws of thought lie in the germ of the simplest and most familiar act of thought. Logic is not accordingly a science of abstractions; it is a science of what is real; of that than which nothing to human thought can be more real; a science of absolute fact, of a fact of observation. Most true is it that when speculation in its studies strays away from this germinal fact it is sure to run into error. Much more, if it starts from anything but such fact it must be baseless and untrustworthy. This has been, it must be confessed, too generally the mistake and the bane of logical speculation. Because logic is thus a science of the real, the actual in human experience, all its teachings may be tested at pleasure. If anywhere it inculcates what cannot be verified in actual experience, it is thereby convicted at once of error and demands correction.

§ 10. Logical science, still further, is, if any science, what is denominated a *pure* or *necessary* science. In this science, not only does thought appear as its own creator forming a science out of its own substance, and so making logic a self-sub-

sisting science, independent and pure of all foreign matter; not only is it a science of the real, of fact in actual experience, of the real that is nearest and deepest to the human intelligence as knowing in it only itself, founding itself on what is most real and building itself up only with materials which itself furnishes in the varying experiences of human knowledge: it is in a higher sense than can be supposed of any other science, a pure or necessary science in that its every legitimate result and development is one that must be admitted. The primary fact in experience, the fundamental thought, as being a fact of observation, has indeed the character of contingency that attaches to every fact outside of thought itself. Logical science must begin so far with what is contingent. But with a thought given, even in its extremest simplicity, after this, all logical science deals with nothing that is foreign to thought itself, and therefore its results are all necessary to thought in the sense that thought in questioning them would sell its birthright to the sole inheritance of truth; would in fact question its only life and nature. The denial of any legitimate outgiving of perfect thought is in fact a suicidal act to thought, to knowledge, to truth. Logical science accordingly is in the highest possible sense a pure or necessary science. Mathematics is to be esteemed such a science only because it is founded on logic—on what the science of logic sets forth and expounds as a property of pure thought. Out

of this property—that of quantity, or more exactly out of thought regarded as quantitative in one or the other of the two kinds of quantity indicated in logic—intension and extension—mathematical science, moving in the exact lines prescribed by logic, works out results that cannot be questioned and so entitles itself to be ranked as a pure or necessary science. Its fundamental *data* are derived from thought as unfolded in logic. It rests on them, not on assumptions, however denominated, whether as definitions, or axioms, or otherwise. And just because it rests on this sure foundation of thought or knowledge and moves in the logical lines of perfect thought, it has its high rank and sway. Other sciences, founding on matter that is foreign to thought, participate more or less in the character of contingent science. As the movements of pure thought in them prevail and give character to the development, the science comes nearer to the rank of a pure science; and on the other hand, as the matter of which they treat predominates they fall into the rank or condition of contingent sciences. In accepting the teachings of these contingent sciences, it becomes more necessary to take into full consideration the fact that the very *data* on which they build are contingent—subject not merely to what little contingency may be supposed to attach to logic itself as founding upon fact in experience, but especially as founding on matter that is foreign or extrinsic to thought.

§ 11. While through its subject-matter logical science thus presents a pre-eminent claim to the interest of every reflecting mind, bringing to it that which is nearest to it, more akin to it, participating more largely in its own nature than the subject-matter of any other of the human sciences, through its *form* it presents still additional claims. It possesses this great peculiarity that in it, thought deals with thought alone. Nothing foreign comes in to disturb its own simple method and way, to distract or confuse its purpose and aim. It marks out its own course, defines its own field of view, circumscribing it with its own limitations. It begins, as has been already observed, with a germ of the greatest simplicity; it develops itself by its own laws; it fills out its own ideal. It knows where it begins, how and why it develops its branches, when and where it has completed its growth. It is throughout a work of organic orderliness and fullness. Every ramification is in its place and in its organic relation to the whole and to every other member. Nothing can be conceived more perfect in form than that which logical science shapes for itself as its ideal. Given at the start its germ of a thought taken up anywhere in the field of human experience, it finds in that the type of every conceivable instance of thought, as a botanist would cull a spear of grass at random anywhere from a wide spread lawn, and know that in it he has the type-form of every other spear in the field.

In that typical thought, although the simplest and most familiar, with a greater and juster assurance than any naturalist in any specimen of external nature's products, he finds the essential nature of every thought; and hence the laws or principles regulative in all true and genuine thought, and all the generic forms possible to thought in the infinitude of its lower modifications. We need to dismiss from our conceptions of a true logical science any notions we may have gained in whatever way of fragmentary observations, disconnected processes, uncouth formulas, desultory sketches, fantastic trickeries, and of whatever else is to true culture only barbarism and crudity. The true ideal of logical science, which it prescribes to itself and which it feels itself competent out of its own resources and beyond the disturbance of any other power in science to realize in its completeness, is thus an ideal of a science that is simply perfect in form—a pure idea embodied by its own hand of perfect skill in a fitting body. We must however limit the application of the term 'body' here to the purely inward mental or spiritual. The more external embodiment of the science in word, in language, which is imperfect in itself and but imperfectly admits the forms of thought and so can only defectively express them, so far mars the otherwise pure form of logical science. The conception of logic as it exists as pure thought back of this language-form of outer embodiment, is the con-

ception of a science as perfect in form as the nature of man can attain.

§ 12. Secondly, logical science is of eminent utility as a means of intellectual discipline. A perfect system of thought, a perfect logical science, is or should be a perfect type-form for all thinking. The presence of such a type-form even in the mind cannot fail to exert a potent influence in forming the mind to a habit of right thinking. The study of thought too is of itself one of the most invigorating and wholesome of studies for the growing mind. "The retorsion of thought upon itself," says Sir William Hamilton—"the thinking of thought,—is a vigorous thought, and, consequently, an invigorating exercise of the understanding; and as the understanding is the instrument of all scientific, of all philosophical speculation, logic, by pre-eminently cultivating the understanding, in this respect, likewise indicates its ancient title to be viewed as the best preparatory discipline for philosophy and the sciences at large." The study of logic, further, leads the mind into the true ways and methods of thought, and is thus the best guide in its training. The habitual employment of the thinking powers in the legitimate exercises of logical training, forms them to the habit of natural, easy, true thinking, whatever may be the subject to which the mind may apply itself. The science of thought, still further, by revealing the natural forms of genuine thought, gives needed

light to all the operations of thought; not only discloses the path to be taken but gives assurance of its being the right path to truth and knowledge. If it is much to know, it is often more to know that we know, to be reasonably assured that what we accept as true is indeed true. Confidence in the correctness of the procedures of thought, confidence in the validity of the results of thought, is one of the leading conditions of success in all thinking. To be in doubt whether we are on the road to truth, whether we are in possession of the requisite means to attain it, whether what we have at last reached is the truth we seek, is very mental imbecility. Logical science not only thus dispels doubt, but preserves from illusions and phantoms. As ignorance of the principles of physical science betrays the sanguine inventor into costly expenditures of time and labor that prove worthless simply because in violation of natural laws, so ignorance of the simple laws and legitimate forms of thought and knowledge is ever betraying the ambitious and sanguine explorer of truth into speculations often worthless, sometimes baneful, never certain. Moreover, logical science, by giving a knowledge of the relations in things so far as true and of the proper ways of access to them, invites to mental effort. It discloses occasions for thought and calls it into exercise. The chief obstacle to thinking is ignorance how to think. Matter enough is given in every outlook upon

the world around us and the world within us to provoke and sustain constant activity in thinking. Thought does not go forth as this gate of occasion opens to it, because it does not see. Logic is a perpetual prompter to thought. Logical science thus is a model and exemplar of all right thought; a guide to true thinking; a stimulant and aid in all legitimate intellectual exertion and practice.

§ 13. Once more, logical science is an indispensable instrument in the successful pursuit of truth and knowledge. The true in the universe of being is the one object of the intelligence, comprehending all that is to be known. The true, in other words, is the exact correspondent and correlative of knowledge. It is accordingly exactly conformed to knowledge, as object to subject. We think what is true in the universe because the true is itself the outcome and exact expression of thought. The unities in all true subjective thought which logical science unfolds —its identities—it leads us to apprehend as we send out our thought over the reality of object around us. It bids us to seek in our pursuits of knowledge only these, and requires us to hold only to what is embraced in these unities or identities, rejecting all irreconcilable discords, all real contradictions, since whatever is contradictory is foreign to true thought. There is a universe of truth as there is a universe of being. All veritable knowledge is in perfect accord and

harmony. The parts of this one whole of all knowable being and of truth, as parts of one whole, logic teaches us, must be in organic relationship to the whole and to one another; and it opens to us the true anatomy of this whole, indicating the partitions or divisions which properly belong to it, which are natural partitions, and consequently those that are to be followed in a scientific dissection. Logical science thus introduces us into the temple of truth—guides us to it, enables us to identify it, opens its doors to us, and discloses to us all its grand compartments as well as its more hidden recesses, all in their architectural order and relation. We are enabled thus under its guidance to enter intelligently upon our studies, on whatever part we may select, assured of the aim that we propose to reach, understanding whither we go, where we are at each step of our progress, and what as to its particular character as truth or knowledge we ultimately find. Not only this, but it also lends us needed helps and instruments in our studies. The parts of truth as organic are interdependent; and when we have acquired possession of one we have in the acquisition the means of acquiring more. Logical science provides us with the proper keys to unlock the remoter stores of knowledge, as well as instructs us how to aggregate all our successive acquisitions. The treasures that lie hidden in a field we have acquired logic teaches us how to reach

and extract and then how to verify them, each and all, as real gold of science. The members of the one body of truth being vitally connected each with every other, logic teaches us how we may acquire legitimate and assured control over a connected member by a right and dexterous use of a member over which we have gained control. The boundless domain of inductive science, logic thus opens up to our incursions, instructing what of truth we may expect to find, in quality and in extent, as also how; for induction has its principles and legitimate forms established in logical science, as fully and as scientifically trustworthy as the deductive movement of thought. When a fragment of a bone was presented to Cuvier, he was enabled in thought to read in it the size, age, habits, the specific characteristics of the animal to which it belonged, the climatic conditions to which its life was subjected, its environment generally. So thought, out of a fragment of fact, a particle as it were of the great body of truth, interprets rich and vast treasures of knowledge, through this organic relationship existing in all truth. It is most true, as Aristotle insisted, that 'it is the part of experience or observation to furnish the beginnings in regard to each particular science'; but these beginnings are but the beginnings and form but a small part of the body of the science. It is true that logic is not thought; it is true, also, that men think without logic. So mathematics is not astron-

omy, and men can compute without arithmetic. But how extremely limited and uncertain is all such unscientific computing and thinking? For the investigation of truth and the discovery of fresh knowledge as well as for the correction of errors in attained knowledge, for the advancement and perfecting of knowledge, and also for the validation and demonstration of truth,—the testing and proving of knowledge—logic is an indispensable instrument. It alone can give the light which intelligence demands as a condition of its progress and its safety, in all the realms of knowledge in every particular science alike.

§ 14. But logic stands in a peculiar relation to two leading sciences, where its help is more indispensable to any assured success. They are the sciences of language and of metaphysics or philosophy.

In regard to the last named science, metaphysical or proper philosophical science, it has been too truly remarked that, "as a system if it does not contain a logic, it will lead to the results of most systems of metaphysic, that is, to none at all." The philosophy of being cannot but prove baseless and untrustworthy that is not founded on the philosophy of thought. Only as thought is understood in its essential nature and forms, and its legitimate movements ascertained and defined in themselves and in their relations, can there be any true and assured philosophy of being. Of course without this under-

standing there can be no proper science of the relations of thought or knowledge to being. What knowledge is and how it is to be identified, must be determined before the first step can be taken in philosophical speculation. And every advance in philosophical knowledge must be along the established lines of legitimate thought, and known to be so. The assumptions which metaphysicians have so generally laid as the foundations of their several systems, beyond at least the simple fact of some actual experience, are utterly unwarrantable in a system of assured knowledge; and the superstructures built up on such assumptions must be unstable and insecure. The sad, yet legitimate result of such baseless, illogical assumptions has been what might rationally have been anticipated, a gloomy, pessimistic skepticism or agnosticism. There is truth, however; and neither the agnostic nor the skeptic can deny or question it. We do know; and we can know that we know. Truth, knowledge, has a nature of its own; has its essential characteristics; and these are as truly open to the inner observation as the characters of the sun to the outer inspection. These ascertained characters or attributes of truth or knowledge are at once its laws and its tests. It is in short little more than a truism to say that the philosophy of being must spring from and out of the philosophy of knowing, and be guided and shaped and authenticated throughout by it.

§ 15. In regard to the other of the two sciences to which logic bears a peculiar relation—the science of language,—as all rational speech is for the expression of thought, logic must be regarded as the parent of grammar. All the forms of language are determined by their use as embodiments of thought. All speech-forms are thus but embodied thought-forms. What forms are required in language, what parts of speech, what functions these parts must respectively perform, what relationships must exist between words and phrases and sentences, it is the business of logic to prescribe. Grammar is wholly and absolutely dependent on logic for this service. Its principles and its proper forms cannot be set forth with any confidence except under the guidance of logic.

But there is another relationship to be recognized between these two sciences; logic is not only the parent of grammar, but logical science is dependent on language for its embodiment and its medium of work. Thought is well nigh impotent except as it can support itself on fitting words. Logic is under the necessity accordingly of considering the function and uses of language in the embodiment of its forms. It needs language and can hardly begin to move without it, and uses it as a most effective instrumentality in promoting its proper work; and what is more, is liable to be misled, to be misrepresented, to be betrayed into errors and doubt, by the im-

perfections or the misuses of language. Logical science must for its own integrity and success take careful cognizance of speech, exacting ever a legitimate interpretation and forbidding equivocal and illusive expression. But its higher relation to grammatical science is that first-named, that of parentage. In a sense peculiar and higher, the successful prosecution of the study of language and of the advancement of grammatical science, demands the aid and guidance of logical science.

PART I.

THE ELEMENTS OF THOUGHT.

CHAPTER I.

THE NATURE OF THOUGHT.

§ 16. LOGIC, according to our definition, is the science of thought (§ 1). Its one fundamental postulate and the condition of its being is this its subject-matter—human thought. But it is thought as the one form of a proper knowledge —thought as a full and completed cognition. Thought or knowledge must have an object and this object must be presented to the mind to be apprehended by it; and this apprehension of the object by the mind may, in psychological analysis, be distinguished as the presentative stage of a cognition or knowledge. As soon as the intelligence begins its proper action on this given object thus apprehended in reflection, the second stage is reached which leads on to the form of a completed thought or knowledge. It is of this completed thought, this proper reflective knowledge, as distinguished from a mere perception or

intuition, that logical science treats. But it is only the pure thought, the thought as abstracted from the object presented and apprehended, with which logic has to do. In founding itself on what is real, on a fact in actual experience, in other words, on some instance of thinking, it is a matter of indifference what particular instance be taken as the norm or type-form which shall exhibit the proper nature of pure thought. The entire essence of pure thought must be found in every instance of proper thought, whatever be the object thought of, whatever the end or purpose of the thought, whatever its degree of complexity or comprehensiveness. We will take for our inspection the simple thought: The sun is bright. The sun presents itself to our sense of sight and impresses it; the mind receives the impression thus, apprehending the object singly and simply as bright, in proper perception. The reflective intelligence—the true and proper intelligence—now exerts itself on this perceived object, which exists as presented to it only as a peculiar impression upon it, by recognizing it as a thing that is bright, and comes to a full and completed knowledge that may be expressed in words: *this thing—the sun—is bright.* The simple phrases, *brightness impressed, sun bright*, expresses no proper thought or knowledge. Not until the intelligence affirms in full sentence, *the sun is bright*, is a true and full cognition or knowledge attained. The one *datum* demanded for logical

science is found in this proposition. It contains the entire essence of pure thought.

§ 17. In reflective analysis of this thought, expressed in this proposition or sentence, we discover at once three distinguishable elements: First, that of which we think—*the sun*, called in logic *the subject;* Secondly, that which we think of the subject—*bright*, called the *attribute* or *predicate*; and Thirdly, the asserting, affirming element *is* which connects the attribute with the subject by identifying them as belonging to the same one object given to be reflected on or thought. This element is called *the copula.*

This act of the intelligence is technically denominated *the judgment.* Expressed in words it is known in logic and rhetoric as a *proposition*, and in grammatical science, as *a sentence.* As has been indicated, the judgment is the one full and complete product of the intelligence; the one full and complete cognition as a form or stage of knowledge.

The subject and the predicate of a proposition are together denominated *the terms;* the proposition itself being viewed as a line of which the two terms—*termini*—are the bounds or limits of the proposition. The subject of a proposition or thought may be defined indifferently as *that of which something is thought* or as *that to which something is attributed;* and the attribute or predicate as *that which is thought of a subject* or as *that which is attributed to it or predicated of it.*

Both subject and attribute—*sun* and *bright* in the proposition—we have found in the same one object given us in the single impression on the mind. They are given as one and the same, entirely indistinguishable in the impression itself. It is the reflective act which makes a double out of the single impression, and its entire activity is expended in first distinguishing the single into a double and then affirming their identity. Its sole function is thus consummated in the identification of the subject and the attribute. This identification is expressed in the copula *is*. The copula thus expresses the vitality of a judgment, and its essence is simply identification,—identification of subject and attribute as distinguished by the preparatory act of analytic reflection in the same one object presented to the thought. The copula may be defined as *the assertive element of a judgment*. It imports simply *a sameness or identity between the subject and the attribute*.

It will be remarked that the identification in an act of thought does not necessarily import a full and complete sameness in the subject and attribute. It is in fact most commonly a partial sameness only that is asserted. In the instance taken, the subject—*sun*—is only in one respect, as to one of its attributes, the same as the predicate—*bright*. The identification contained in a judgment may thus be *total* or *partial;* while most commonly in actual uses of thought, only a

part of the attributes are included in the identification. The more precise consideration of the relations in respect to quantity between the terms of a judgment will have its place appropriately in a subsequent chapter.

For the purpose of simplicity, we have spoken only of the positive or affirmative action of thought. The human intelligence, as an assertive function in relating attribute to subject, is as competent to deny as to affirm; to disconnect as to connect subject and attribute; to difference darkness from the sun, as identify brightness with it. A judgment may be thus either affirmative or negative.

The full nature or the entire essence of a proper thought is thus found to consist in the identifying or differencing of attribute with subject. We have thus the definition: *thought is attributive cognition.* Or if the conditional and preparatory stage of thought be embraced in a fuller and looser use of the term, this thought will be exactly and fully indicated in the definition :—*Thought or knowledge is apprehensive attribution.*

This exposition of the essential nature of thought, if correct, must clearly be regarded as the one germ of all true logical science, the one foundation on which the entire superstructure rests. As such it gives the one clear light which illuminates the whole development of the science, exhibiting the exact import of all its particular

teachings, at the same time that it demonstrates the truth of all and invests them with their high authority over all the realms of true knowledge.

§ 18. Of the correctness of the exposition, it would seem, there can be little reason to doubt. The simple inspection of a judgment demonstrates it. When we think *the sun is bright*, on the simple apprehension of the object through the sense, we do no more and no less than recognize brightness as an attribute of the object regarded as a subject, that is, as having attributes. In other words we recognize the complement of attributes that make up or constitute the sun as being the same as bright in respect to one—to a part of that complement. This is the sole import of the proposition: *the sun* as a body of attributes is the same in respect to one of them as *bright*.

The simplicity of the exposition commends it. So also does its perfect and ready applicability to all the manifold complications and modifications of thought.

Psychologists recognize attribution as the constituent of all thought. In thinking we do nothing more or less than attribute some property or relation to some object; and this object to which the property or relation is ascribed is held to be the subject in the attribution. Psychologically, attribution is but the recognition of the sameness of a subject with its attribute in whole or in part.

Logical science has tended to this view; has

recognized it, here and there, in its teachings, although omitting to exalt it into its due place as the germinant principle in all logical exposition, and viewing it only in partial applications. It is sufficient to cite the authority of Sir William Hamilton, who declares unqualifiedly "that a proposition is simply an equation, an identification, a bringing into congruence of two notions in respect to their extension." His limitation to extensive quantity is without ground. Indeed the statement in its form indicates that the truth had not been precisely thought out in his own mind, although unhesitatingly accepted. But particularly in their enumeration and exposition of the fundamental laws of thought logicians have fully accepted the truth that identification between subject and attribute is, with its necessary correlate and complementary differentiation, the principle of all affirmation and all negation —of all thought whether affirmative or negative. The symbolical and algorithmic systems of logic assume this truth as fundamental.

CHAPTER II.

THE FUNDAMENTAL LAWS OF THOUGHT.

§ 19. THE fundamental laws of thought must be rooted in the essential nature of thought and be the exact expressions of that nature. Only so can they have authority over thought, or become regulative and validating principles of thought. The sovereignty over thought cannot be imparted from a realm foreign to it, much less can any mere assumption, any ungrounded, underived principle be accepted as a fundamental law of thought however clear in its statements or unquestionable in its truth. We can accept no thought to be a genuine, a true thought unless it be seen to possess the ascertained nature of a thought; and no rule can be regarded as authoritative in the last resort except as it is invested with the character of a genuine offspring of thought. We must seek in thought itself the ultimate source of all authority and rule in it.

The essential nature of thought we have found to consist in attribution; a proper definition of thought is *attributive cognition*. Only as we recognize an object in the character of a subject having or not having some attribute have we a cognition that can be called proper thought—a

reflective cognition as distinguished from mere mental apprehension. All attribution is either affirmative or negative; it identifies an attribute with a subject or differences an attribute from a subject. If we think at all, therefore, we must attribute—we must identify or difference some attribute with reference to some subject. And as simple thought consists of nothing else but such affirmative or negative attribution, we must in thinking do nothing else, as we should in such case either miss of thought entirely or vitiate the purity of thought and so make our thought, so to speak, false thought.

We have thus given us two fundamental laws of thought, of absolute authority, springing from the essential nature of thought: First, if we would think at all, we must attribute affirmatively or negatively; secondly, we must not in thought do anything else than thus attribute. These two laws have been known, the first under the name of the *Law of Disjunction*, and the second under that of the *Law of Exclusion*, more inadequately and in special application sometimes called the *Law of Excluded Middle*. These two laws are obviously the most fundamental laws of thought that we can conceive. They possess the most absolute authority; and this their authoritative character they derive directly from the nature of thought. They are not assumed; they are in no proper sense *apriori;* they are not properly *first* truths, much less innate principles.

They are, moreover, not axioms or underived principles, but are immediately derived from actual inspection of an act of thought—from a fact of experience. They accordingly participate from their very origin in the nature of what is real or actual. These two laws, still further, form a pair of laws which are in the contrasted relation to each other of positive and prohibitory laws. The law of Disjunction is positive, requiring of us, that if we think at all, we attribute. The law of Exclusion is prohibitory, forbidding us to do anything else in thought but attribute.

§ 20. But, farther, we need to think rightly; and to do this we must, in the first place, attribute rightly, that is, in attributing, we must identify with a subject some attribute actually belonging to it or difference from it some attribute not belonging to it, and in the second place, must not do the contradictory of this, attribute to a subject an attribute not belonging to it or difference from it an attribute belonging to it. This is evident from the very nature of right thinking. We have thus two other fundamental laws of thought: First, the *Law of Identity*, requiring us, if we would think rightly, to identify with a subject an attribute actually belonging to it or difference from it an attribute not belonging to it; secondly, the *Law of Contradiction*, forbidding us to identify with a subject any attribute not belonging to it, or to difference from it an attribute actually belonging to it. We have thus

another pair of laws, one positive and one prohibitory, founded clearly in the very nature of right thinking, and thus invested with absolute authority over all thought; ascertained too by mere inspection of right thought, not assumed, not *apriori*, not innate, but founded in an observed fact of experience and so possessing a truly real or matter-of-fact character.

It is obvious that these four laws are the only laws of this order. The enumeration is exhaustive in reference to their source or ground—the essential nature of right thinking. They are of equal authority and rule with absolute sway throughout the realm of right thought. They will of course suffer diverse modifications in their respective applications to thought as diversely determined by its subject-matter or by its end. All possible forms of thought must be evolved under their regulative sanction and be subject to their testing and validating authority. They appear, as we have seen, in two pairs or duads, each pair comprising one positive and one prohibitory law. They justify repetition, and may be most correctly arranged in the following order, viz.:

FIRST LAW OF THOUGHT:—*The Positive Law of Disjunction*, requiring us, if we would think at all, that we attribute either affirmatively or negatively. This law is positive. It involves the necessity, in order to the form of thought, of the alternative of affirming or denying—of identifying or differencing. It implies in any form

of thought the truth of one or the other branch of this alternative—*is* or *is not*—*this attribute does* or *does not belong to this subject*.

SECOND LAW OF THOUGHT:—*The Prohibitory Law of Exclusion*, which forbids us to do anything else in our attempt to think than to attribute affirmatively or negatively.

THIRD LAW OF THOUGHT:—*The Positive Law of Identity*, which requires us, if we would think rightly, to identify with a given subject an attribute belonging to it or difference from it an attribute not belonging to it.

This Law has been also called the Law of *The Same and the Different*.

FOURTH LAW OF THOUGHT:—*The Prohibitory Law of Contradiction*, which forbids us in all right thinking to identify with a given subject an attribute not belonging to it or to difference from it an attribute that actually belongs to it.

§ 21. It would be difficult to trace the history of the enunciation of these fundamental laws of thought back to the absolutely primitive source or author. In fact, their first appearance is, as would naturally be supposed, only incidental. They appear in form as a part of logical teaching only in the most recent times. The principle of Identity, it is supposed, may be discovered in an utterance of Parmenides, a Greek philosopher of Elea, who flourished in the middle of the fifth century B.C. In a poem on Nature, only fragments of which remain, he says: 'We must say

and think that *what is, is.*' Plato and Aristotle in their philosophical works over and over reiterate the principle, but in no formal way. The law has no place in the Organon of Aristotle, the nearest approach to it being the assertion that "all truth must always be consistent with itself." The negative correlate of this law, the law of contradiction, is in like manner recognized in a passing way by these early thinkers. In the Organon of Aristotle, it is once appealed to as a determinative principle; 'for it is impossible that the same thing at once be and not be.' The principle of Disjunction, or that of alternative affirmation or denial, is incidentally enumerated in the Analytics with other truths commonly accepted as self-evident and common to all sciences, such as "Equals from Equals" and the like. That of Exclusion is repeatedly recognized in his Metaphysics, as, B. III. c. 1, "no middle is admissible in a contradiction, but it is necessary either to affirm or deny any one attribute of any one subject whatever." In his Posterior Analytics he defines Contradiction as "opposition which in respect to itself has no middle."

Only in the more recent logic do we find any formal enunciation of the laws. German systems have commonly enumerated four. They combine those of Disjunction and Exclusion, as given above, under one called Excluded Middle or Excluded Third, and add as a fourth, the Law of

Reason and Consequent. But evidently there is as much reason for separating the laws of Disjunction and Exclusion as those of Identity and Contradiction. All logical considerations, as Exactness and exhaustive Thoroughness, as also Consistency and Symmetry, prescribe clearly the fourfold enumeration we have given—two pairs or duads, each with one positive and one prohibitory law. The Law of Reason and Consequent, Sir William Hamilton, who in the text of his Lectures on Logic had accepted it from the German system, afterwards very justly rejected as only a corollary from the other laws. It is in fact a mere assumption. It has no discernible ground in the nature of thought. It stands in no logical system with the others. In short, it is utterly unworthy to be regarded as a fundamental law of thought.

The number, nature, origin, validity, authority, and utility of these laws have provoked extensive discussions among logicians, and especially in the last half century. The disagreements have been largely occasioned by disagreeing interpretations of language, which as used has been often defective or erroneous, yet sometimes, in legitimate usage, ambiguous. It will be of use to note some of these sources of disagreement.

The term *law* has been diversely interpreted and diversely used. It has sometimes been viewed as a kind of ordinance issued from some sphere extraneous to thought authoritatively con-

trolling its processes. Sometimes, it has been regarded as an innate principle in the human mind, appearing there in some mysterious, indefinable way, or as a first truth of the origin of which no account can be given or reasonably demanded, an absolutely *apriori* principle, that must be assumed. But by a law of thought can be properly meant only an essential attribute of thought which must of necessity exist in all genuine thought, just as gravitation is a law of matter, because, an essential property of matter. A law of thought can be said to be innate only in the very narrow import, that man is born a thinking being, and if he thinks and thinks rightly, his thought must have certain essential properties. These essential properties regarded as governing readily take to themselves the denomination of *laws*. A fundamental law of thought is *apriori*, at most, only relatively and in the sense that the attribute which it represents must actually be a constituent property of all right thinking. These so styled laws are by no means principles existing extrinsic to thought that come in from without to dominate it. Much less are they to be deemed underived, original, independent, self-subsistent principles. They are, as we have seen, derived directly from thought itself and are discernible in any instance of thought, just as gravitation is discernible in any material body. Thought exists, has certain essential characters or attributes that constitute it, or make it to be

such; these are the laws of thought, and can no more be conceived of as existing independently of thought than gravity independently of matter. If the terms *apriori*, *innate*, *first*, and the like, were confined when in use to this relative signification, which is the only one admissible in this application, much of the apparent discord would disappear. Farther, these laws of pure thought have sometimes been interpreted to be laws of existence properly objective to thought, and not as they should be, simply subjective laws. Thought implies an object; and every such object must be thinkable, be such, in other words, that the human mind can apprehend it and recognize it as a subject with attribute. But as to the specific content or nature of any attribute apprehended from without to be thought, the laws of pure thought can have nothing to do in the way of determining it. The laws of nature are not the laws of thought, although the laws of thought are the laws of every thinking nature. Once more, the laws of thought are not to be regarded as principles which are to be distinctly recognized in consciousness so as to be consciously applied in every exertion of thought. The conscious observance of rule is not always necessary, and for the most part only distracts and hampers exertion.

The explication of these laws, moreover, has been such as to mystify and repel. The law of identity has thus been explicated as meaning

simply *whatever is, is.* This form of statement would naturally be interpreted as respecting the content—the object—of thought, not thought itself. It confuses the being of thought with the being of objective reality—the almost universal, yet most baneful fallacy of logicians. Besides, it strips the law of its proper logical significance and makes it unavailable for use. Much the same must be said of the formula by which some have chosen to represent it—*A is A*, or $A = A$. But, as has been shown, the law expresses properly only the one essential element of all right thought or attribution, that in the positive forms the attribute belongs to the subject, that is, is identical with it as a part with its whole. The law is thus seen to be simple, necessary, unquestionable, and readily applicable to use, first in the judgment and then in all the derived forms of thought; for all thought, in whatever form or application, or however fragmentary, must possess this essential character of thought, that of attribution, which is but identification of attribute with subject. The law has thus a most significant and extensive application in regulating actual thought.

Analogously to this, the law of contradiction has been explicated in the formula: *nothing can be and not be.* The formula has an objective, not a proper subjective aspect. It strips the law of its proper significance and availability. The same may be said of the explication in the for-

mula: *what is contradictory is unthinkable*, a — *not-a* $= o$, or $a - a = o$. If on the other hand the law be explicated as simply the prohibitory form of the law of all attribution which in its positive form identifies attribute with subject, it is seen at once to be most significant and available to use in respect to all the forms of true thought primitive or derived, as well as also of unquestionable legitimacy.

In like manner, the law of excluded middle, as expounded in the formulas, *everything must either be or not be; A either is or is not; A either is or is not B*, loses nearly all its significancy and availability for use, and appears as a groundless assumption, not to be questioned, indeed, and precisely because it is little more. at best, than a truism. Another grave objection to the expositions given of this law, is one already intimated, that it blends the two forms, positive and prohibitory, so as to make the exposition obscure and altogether inadequate as a law of thought. The name *excluded middle* restricts its applicability to a single form of thought—the syllogism. It cannot be vindicated except as a mere assumption with no authority except this, that no one can question it. The true exposition of the twofold law—Disjunction and Exclusion—as given above, indicates at once the necessity of recognizing the distinction into two laws—one as positive, the other as prohibitory—and the diverse applicability of the two is obvious. While each

has its application in its own way everywhere in thought, the first named—Disjunction—both prescribes what we must do if we would think at all,—identify or difference attribute with subject, and also gives authority to the inference that so far as the essential form of thought is concerned, the one or the other—identifying or differencing—must be true, which is the validating principle of the disjunctive judgment.

The disputes that have arisen over these laws, as well as the utter rejection of them by some writers, are to be ascribed, in part at least, to these erroneous explications of the laws.

The validity and authority of these laws depend on this simple question: are they truly essential attributes of thought? Has thought been accurately observed, and are these attributes actually discernible in it? The laws of thought are attained in no other way than by such actual inspection of thought. The only question consequently that can arise as to their validity and authority, is: is this inspection accurate?

§ 22. The utility of a well grounded complement of the laws or essential attributes of thought presented in a formal, systematic enunciation, are too obvious to require extended discussion To such an enunciation certainly no objection can be made on the ground that it is not eminently rational and proper in a logical treatise. But it is positively needful to logical complete

ness. It will be found moreover most helpful to the study of the nature and right use of the different forms of thought. It furnishes the means of testing as well as directing specific thought; giving assurance to all thought conformed to their requirements and making it possible to reject with indisputable authority all forms or pretenses of thought not conformed to them. These laws of thought, in fine, are not only indispensable in any worthy science of thought, to any satisfactory exposition of the nature and validity of the different forms of thought, but will be found most serviceable as guides and helps in actual thinking,—in the acquisition and validation of truth and equally in the detection and exposure of error.

CHAPTER III.

THE JUDGMENT.

§ 23. WE have found the essential nature of thought to be attribution, or, more precisely, conscious attribution, in which an attribute is identified with some subject or differenced from it (§ 17). This conscious attribution is known in logical science under the technical denomination of a *judgment*.

A JUDGMENT may accordingly be defined as the *product of that mental act in which an attribute is affirmed or denied of a subject*.

A logical judgment accordingly is a product of mind considered apart from the body of words in which the thought is expressed. A judgment thus embodied in language is, as already stated, called a proposition, or in grammatical science, a sentence. A judgment is Aristotle's inner rational product or *logos;* a proposition is the uttered *logos*. A science of propositions is hence a very different science from that of mere thought. J. S. Mill, in his "System of Logic," which is a science of propositions, and not of thought, departs from the views of the great mass of logicians, who, as he truly says, have regarded it as "a theory of judgments."

§ 24. The definition sometimes given of a logical judgment and made the basis of theoretical expositions in both logic and linguistic science, as well as elsewhere in metaphysical speculation, that 'it is a subsumption of a singular under a general,' is accordingly to be entirely rejected. In the first place, it implies that the general can subsist before the singulars of which it is but the aggregate, and therefore a logical posterior and consequent. In the next place, it recognizes but one of the two great species of logical quantity. Moreover, it is wholly inapplicable to a large number of judgments.

In like manner the definition given by Ulrici that the logical judgment is 'a differentiation,' is to be rejected on the ground that it is partial and covers but a single class of judgments. We have identifying as well as differencing judgments. The former must indeed be admitted to be the higher class; and identification is as truly and as fully implied in a differencing judgment as the latter in the former. The proposition was more justly defined by Aristotle as including both the being and the not-being of an attribute in a subject; identification and differentiation. The positive or affirmative must be supposed to be prior in nature to the negative or differencing. We naturally fix in our thought the positive contents—the essence—of a thing before we bound it by separating or differencing it from what is outside. We recognize the internal characters

before the external—the properties before the outer relations.

A judgment, further, is not accurately defined as a comparison, for comparison is not of the essence of a judgment, however much we may compare in forming a judgment. Much less does it consist, as Jevons teaches, in comparing together two notions, "so as to ascertain whether they agree or differ." The inspection of the agreement or difference and also the comparison may precede the judgment. They may take place without being followed by an actual judgment. They do not constitute the essence of the judgment.

Still further, a logical judgment does not, as some have held, necessarily involve the recognition of reality or existence in the matter of the judgment. The being of thought and the being of reality—actual existence—have been confounded very commonly and very unfortunately both by logicians and by metaphysicians. But reality is an attribute, which is to be discerned like any other attribute. It must first be apprehended as given or presented to the thinking function, before it can be thought. We judge in matters of mere imagination, of purest fiction from which all reality is excluded. We recognize also a realm of *abstract truth*, into which nothing of reality, of actual existence, enters. We judge thus that a square figure is quadrilateral and rectangular without thinking of any outer existence,

and even independently of a diagram. The thought, the judgment is as genuine in such abstract matter as in real things. The thinking act is real; the matter thought is not always and necessarily real.

§ 25. The judgment must be recognized as the product of a true life—of the mind as a truly living factor. It has accordingly an origin, a rise, a genesis, conformed to the nature of the products of all living things. It bears, indeed, a relation to its immediate source—simple apprehension, in the form of a perception or an intuition—similar to that which a flower bears to a bud. It springs directly from the simple apprehension. It cannot exist, except on the condition of the *datum* or matter, the reception of which in the mind constitutes the mental state termed *apprehensive*. It is completely bounded by that, so that a fundamental principle regulating it is that the judgment can never transcend the matter originally presented in the *datum*. As the product of a life, its rise or genesis from the simple apprehension is a proper life-process and necessarily continuous, as the bud by a continuous movement passes into the flower. As the bud and the flower are two stages of plant life, so perception or intuition and the judgment are to be viewed as two stages of intellectual life rather than as two kinds of cognition or knowledge. The full, mature, or completed knowledge is reached only in the judgment. Every full and complete knowledge, more-

over, is a logical judgment. So that we exactly define a judgment to be a full and complete knowledge or cognition.

§ 26. The genesis of the judgment exhibits its true relation to the matter or the object which is thought in the judgment. To exemplify this we may more conveniently, as before, take a judgment — a thought, a knowledge, a cognition — respecting some sensible object — as *the sun*. Through the interaction existing between the organic parts of the universe of being, the sun impresses our sense—say, our sense of sight—causing in our minds the impression of *bright*. On this condition of the mind, which, as we must conceive it, is an impression on an active nature indeed, but rather determining the passive than the active condition of the mind, the purely active mental faculty of knowing immediately exerts itself. Thought acts on the received impression, not directly on the impressing object. Every judgment, every full and complete knowledge or cognition thus respects immediately the mental state; the perception, as we call it, the simple apprehension, not the external object itself, not the sun in the heavens, but the impression produced by the sun in the mind's receptive or apprehensive nature. The thinking or cognitive faculty performs its own proper function now by seizing, in this object thus impressed and apprehended, some attribute as a part, an element, a characteristic of it, and by holding it as belonging

to the impressed or apprehended object, in other words, as identical, so far as such part, with it.

§ 27. The product of this judging act, this thinking or knowing, is sometimes called a *truth;* and the term *truth* is a familiarly recognized synonym of the terms, *thought*, *knowledge*, *cognition*. It differs from these terms as it points outward to the object judged, or thought, or known, while they point to the subject thinking or knowing. But it has itself divers specific uses or modifications which it is important ever to discriminate. It is used thus sometimes more in reference to the relation between the apprehension—the perception—and the object apprehended—*the sun*. The impression of the sun on the sense may be more or less imperfect, defective, distorted. We say the impression is not a correct one—is not true; and so the judgment founded on the impression is not correct; is not a truth. Or, again, the judgment itself may in its own production be disturbed or weakened or obscured; and so mistake may arise as to the quality or the quantity of the attribute seized and identified as organic part of the impression. Still farther, we think mostly in words; and words are far from being always exact embodiments of the objects or qualities which they represent. Thus the judgment, though true in itself, as expressed in language—the proposition—may not be a truth. We have accordingly the same term—truth—predicated first of the relations between the re-

ceiving sense and the object; secondly, of the interior relations of the judging act itself, and thirdly, of the relations between this act and its expression in language. We have truth of apprehension; truth of thought; and truth of verbal expression. The term truth, it may be added, has still another modified use, as it is employed in a mere objective relation, to denote the congruous union of the parts in a given object as a whole. We speak thus of a *true sun* when we mean a sun that has the actual properties or parts of a sun in their organic relationship to each other. Any notion of the sun which should embrace attributes not belonging to it or not in congruous relationship to one another, would be a false, an erroneous notion—not a true notion. We have need then to distinguish these four uses of the term *truth*:—1, truth of apprehension; 2, truth of attribution; 3, truth of expression; and 4, truth in object (§ 189).

§ 28. Once more, we must recognize in this genesis of a judgment or completed cognition the presence ever in it of a *positive* character or nature. It implies an action, the exertion of an active nature. It is as truly active when it denies as when it affirms; when it differences as when it identifies.

§ 29. The judgment is the one only primitive form of thought or knowledge. The perception, as has been seen, is but a germinal, inchoative, incomplete knowledge; is but an incipient con-

ditional stage of knowledge. It is real and so implies time and the capacity of continuance before maturing into a judgment, as also the power of acting immediately on mind and body; it is yet an immature cognition. The concept and the reasoning have been generally represented as distinct kinds or forms of thought. More commonly, they with the judgment have been erroneously treated as three independent and organically correlated forms of thought. Sir William Hamilton incidentally intimates the true relationship between these modifications of thought when he says: "Thought, in all its gradations of conception, judgment, and reasoning, is only realized by the attribution of certain qualities or characters to the objects of, or about which we think." 'Attribution' he here in this passing way recognized as the one essential character of thought in all its modifications; and the three comprehensive modifications he also recognized as 'gradations.' But he places 'conception' before 'judgment' both here and in his formal exposition of logical forms or elements. He presents no formal genetic evolution of one from the other. His language often implies the existence of the concept before the proper 'judgment.' Generally, modern logical systems have treated the concept as prior in origin to the judgment. Of its genesis they make little or no attempt to give any account. Its existence is simply assumed; it somehow appears in the chaos of ideas in human

experience and is identified as a concept by certain relations supposed to be discovered in its elements. Conception differs from perception, concept from percept, as presentative knowledge differs from reflective knowledge; but how it arises, how it comes to differ, no satisfactory explanation is given.

§ 30. The genesis of the judgment, as given above, exhibits the true relationship of the judgment to the concept. The single object, *the sun*, is apprehended in perception, as a simple *bright;* this object single and simple in the apprehension, is now in reflection or thought viewed or known—reflectively cognized, that is, recognized—as a subject with an attribute, as the judgment now emerges which may be expressed in language—*the sun is bright.* Before the judgment there was no concept,—simply the perception, *bright thing.* Now we have in the terms—*the sun* and *bright*—veritable concepts. They are known as such by being either subjects or attributes that may be *terms* in judgments. They both bear the mark of thought—attribution. They are concepts because they have been thought as subject and attribute. They were before the act of thought apprehended only as one; which unit of consciousness has been resolved in the act of thinking into two, that stand in the relation to each other of subject and attribute, each implying the other as a necessary correlative. They are each complete and distinct elements in the judgment. They

emerge into real existence, they come to be, thus, only in and with the judgment of which they are with the copula the members, constituting one organic whole. As the petals of the flower do not exist before the flower, but in it and with it, so the terms and copula of the judgment do not exist before the judgment, but only in it and with it. Such is the law of every living product. Universally, the constituent members of an organic whole can come to be only in and with the organism itself.

Every aspect of the concept confirms this view of its relations to the judgment. The concept is everywhere recognized as a product of thought, of the reflective intelligence. But thought, as in Kant's summary statement, is 'knowledge through concepts.' All operations of the understanding or reflective intelligence of thought accordingly, can, he teaches, be traced back to judgments, so that the understanding may be represented as faculty of judging. There is no thinking or knowledge without judging, and all thought is through concepts. There seems to be certainly some lack of thorough logical consistency in the doctrines of Kant; but these particular views are with him fundamental and unquestionable; all thought is judging, and all judging is with concepts. The concept as a product of thought is referable to the judgment, and consequently is presupposed in the judgment which yet is effected only through concepts.

Such representations can be made only of an organic product in which the whole body and the members come into being together, ever co-existing and ever presupposing each the other.

This exposition will of course be interpreted as referring to the primitive thought to which a single object or simple attribute is presented as its *datum*. The genesis of the concept may always be traced back in respect to its primitive origin to the judgment in which it appears as a necessary constituent member. It may however, once produced, live on in thought, as a separate existence. *The bright sun* is the verbal expression of a concept which may be employed as a term in a new judgment, as *the bright sun is in the heavens.* But I can say *bright sun* only as I have previously attributed brightness to the sun,—only as I have previously judged expressly or impliedly: *the sun is bright.* The primitive concept ever appears only in and with the judgment; and equally the primitive judgment is effected only in and through concepts.

It has been a common error with logicians to regard the concept as essentially composite; as, in other words, embracing a plurality of individuals or subjects, or a complement of attributes. This is an error; for obviously in any primitive judgment, and accordingly in the primitive concept, the subject—*the sun*—is single and the attribute—*bright*—is simple. But even this view of the concept, inadequate as it is, involves the pre-

existence of judgments in which each individual combined into the class is united with the common attribute or each simple attribute is united with the common subject. All generic concepts and all comprehensive concepts, into which logic has inadequately distributed all concepts, imply accordingly a previous attribution or judgment in each particular constituent of the composite concept.

Besides the consideration that except as organically co-existent with the judgment no satisfactory explanation can be given of the origin of the concept, there is the related truth that only by a reference to the judgment can the concept be tested as to its validity. If concepts are thus precedent to all judging attribution, then all thought, all knowledge, and all truth, must rest on the concept as its lowest foundation; and if there be no logical basis for this foundation itself in the concept, there can be none for knowledge or truth generally. All thought would be without possible validation.

§ 31. The judgment, we conclude then, is the one fundamental primitive knowledge or thought. To it all other modifications of thought are traceable either as incipient stages or as constituent members. Perception, in the exactest view, as already shown, marks only a stage of knowledge. The concept can be verified as legitimate only as it is recognized to be an organic member of the judgment. The third form of thought, as

usually enumerated by logicians—the reasoning —is everywhere explained as derivative; it is ever a judgment derived from another judgment or from other judgments, or more exactly a movement of thought in which one judgment is derived from another. Only in the judgment do we find proper attribution, in which lies properly all truth or falsehood. In it thought or knowledge first appears as complete. As organic it exists only in and with its constituent members. In relation to these and hence to the concepts, it is truly in a sense primitive, but only so, however, as the body is conceived to be primitive in relation to its members, which can live only in and through it as it can live only in and through them.

§ 32. This genesis of the judgment also exhibits the true rise and function of the so-called logical *copula*. By this word is denoted the proper assertive element of the judgment. It is the expression of the activity of the thinking nature, and so constitutes the more positive element in thought. It positively unites subject with attribute, identifying or differencing them, and is thus the positive element in all attribution. It is the vital element of the judgment in the sense that it is the more active element and expresses the essential nature of the intelligence as activity. Without it there can be no judgment, no attribution, no thought, no knowledge. It is that which makes it possible to predicate truth or falsehood of a form of thought. But it is the vital

element of thought only in the sense that it exists only in and with the terms of the judgment, as organic members of the same one whole. It may be represented as the seat of life to all thought, as the heart may be viewed as the seat of life to the body—the more central principle of thought—and thus the higher, more essential principle. Its modifications determine the innermost and most significant modifications of the judgment, as the terms furnish what may be considered intrinsic but less central and vital modifications.

§ 33. SUMMARY OF THE CHARACTERISTICS OF THE LOGICAL JUDGMENT.—1. It is a positive act of the intelligence.

2. It is the primitive essence and principle of all thought and knowledge.

3. It is conditioned on some object presented to it, called its *datum*. A fundamental principle of all genuine thought, accordingly, is that thought must never transcend its *datum*. Most justly and accurately is this principle enunciated by Prof. Mansel in his *Logical Prolegomena*, p. 47: "The act of thought cannot create its own object; being mediate and representative it requires to be based on an immediate and presentative fact of consciousness."

4. Its essential nature is attribution—identifying or differencing subject with attribute. Hence all genuine thought or knowledge bears the character of being attributive—all so-called

presentative knowledge being more correctly viewed as merely inchoative, incomplete cognition.

5. It is organic, being ever a three-membered whole, constituted of subject, copula, and predicate as essential organs which altogether originate at once as the judgment emerges into life.

6. It receives its properly intrinsic modifications alike from each of its organic constituents—those from the copula being more interior and, so to speak, nearer the seat of life to thought, while those from the outer relationships of the judgment, its source or ground and its aim or end, determine its extrinsic modifications.

See Appendix. Exercises: No. I.

CHAPTER IV.

MODIFICATIONS OF THE JUDGMENT.

§ 34. THE judgment, as the product of an active nature or the result of an action, besides its own essential internal properties and relations sustains also necessary external relations both to the source or ground of the thinking act and to its result or end. It will suffer significant modifications from each of those general characteristics and relationships determining so many generic modes or kinds of judgments. This classification, thus formally grounded on all the attributes, intrinsic and extrinsic, will be recognized at once as exhaustive, so far as the necessary elements that affect the formation of the judgment are concerned. It will also help to a more ready and correct understanding of the proper function and significance of each particular kind of judgment as well as show how needful each is in order to completest fullness and richness of thought, particularly as it exhibits the organic relationship of each special form to every other. Altogether it will render it possible to construct a more scientifically exact and complete doctrine of the judgment as the one perfect product of the human intelligence—the one true form of all cognition—of all knowledge and proper truth.

The intrinsic modifications respect the internal character of the thought in regard to *quality*, *quantity*, and *logical form*. The extrinsic modifications are founded on the principles stated, viz.: *the ground* of the thought, *the end* of the thought, and *the verbal statement*.

I. Intrinsic Modifications of the Judgment.

§ 35. The intrinsic modifications of the judgment respect either (1) the copula, or (2) the terms.

1. Judgments are distinguished from one another in respect to the internal character of the copula, technically called its *quality*.

This term *quality*, it may be observed here, is of frequent use in logical discussions, and is used to signify generally *the intrinsic nature of thought*. As specifically applied to the copula, logical quality signifies the properly assertory or attributive nature of a judgment. But it is also applied to the terms of the judgment. It is accordingly to be recognized as of two kinds, which may conveniently be named respectively: (1.) *Attributive* or *Noematic* quality, pointing to the nature of the proper thought-element in the judgment; and (2) *Conceptual* quality, pointing to the nature of the object-element in thought.

The first of these two kinds—attributive or noematic quality—founded on the copula or

thought-element in the judgment—is subdivided into two lower kinds:—1, *Simple Quality*, and 2, *Disjunctive Quality*.

SIMPLE QUALITY signifies the nature of a judgment as assertory. It is determined by the second pair of the fundamental laws of thought named in § 20—the laws of Identity and Contradiction.

Inasmuch as in all right thinking we either identify or difference, we have the two species of judgments, the *Affirmative* and the *Negative*.

AN AFFIRMATIVE JUDGMENT identifies an attribute with its subject; as, *A is B; Venus is bright; The sun shines.*

A NEGATIVE JUDGMENT differences an attribute from its subject; as, *A is not B; Venus is not now visible; The sun does not revolve around the earth.*

The negative quality of a judgment is properly expressed by any of the negative particles in language, as *not, never, nowhere, in no wise*, etc. The responsive *no* is an abbreviated expression of a judgment denying the judgment put forth in the question; as, "*Is truth ever inconsistent with itself? No.*" In a proper negative judgment the negation must be connected with the copula. Hence: "*Falsehood is inconsistent with itself.*" "*He was a non-combatant,*" are not strictly negative judgments.

§ 36. DISJUNCTIVE QUALITY signifies the nature of a judgment as but alternatively identify-

ing or differencing. Such a judgment is called a DISJUNCTIVE JUDGMENT, which may accordingly be defined to be a judgment in which the attribute is alternatively either identified with the subject or differenced from it; as, *A is either B or is not B; The sponge is either a vegetable or is not a vegetable.* It is determined by the first pair of the fundamental laws of thought named in § 20,—the laws of Disjunction and Exclusion.

The very nature of thought, as we have seen (§ 19), shuts up to the alternative, if we would think at all, of identifying or differencing subject and attribute. In doing either we think; and in thinking we can do nothing else than thus to identify or difference; there is no third thing possible. If accordingly in a given case identifying attribute with subject will not give true thought, differencing the attribute from it must. The nature of thought, therefore, through its two laws of Disjunction and Exclusion, both shuts up to this alternative disjunction, and also certifies the one or the other to be true thought, as otherwise legitimate thought might itself produce that which is not true. In the very nature of thought, thus, is grounded the principle of alternative disjunction that any attribute may be thought in relation to any subject alternatively, either as belonging to it or as not belonging to it; one or the other of these alternatives must be true, and if one is true the other is not true.

This characteristic in thought logicians have recognized under the name of *Opposition.*

§ 37. LOGICAL OPPOSITION accordingly signifies the difference implied in the disjunctive quality of thought.

Logical Opposition is pure as it is referred to the copula or attributive element of the judgment. It is in this application called *Pure Contradictory Opposition*, or simply *Contradictory Opposition.* Logical opposition is however conveniently applied to the terms, that is, the matter, of the judgment. It is in this case called *Contrary Opposition.* It is sometimes, when approaching nearest to the character of pure thought, more loosely called *contradictory* opposition. In this case the opposition proper to thought itself is called by distinction *pure contradictory opposition.* But the fundamental distinction in the two kinds should be recognized in this, that the one kind lies in the copula constituent; the other in the terms. The distinction is one of great importance in logical science, but more especially in metaphysical speculation. Aristotle devoted one long chapter, the last of the First Book of the Prior Analytics, to an elucidation of the fundamental difference in the significance of these two kinds of opposition. Nevertheless leading thinkers of the more recent times have fatally to their systems overlooked the distinction. Contradictory opposition, lying in the nature of the thought itself, gives a

necessary truth; as, *A is B or is not B; The atmosphere is bounded or is not bounded.* But the propositions *A is B or non-B, The atmosphere is bounded or it is unbounded*, are not necessarily true. There may supposably be no such thing as a *non-B* thing, or an *unbounded* thing—no subject to which *non-B* or *unbounded* belongs. As Aristotle puts it: "The predications *it is non-white wood* and *it is not white wood* are not the same; for if the wood is *non-white*, it will still be wood; but there is no necessity that that which is *not white wood* should be wood at all." We recognize the fallacy when we attempt to apply the necessities of thought implied in contradictory opposition to such propositions as *the stone is either a vertebrate or an invertebrate; Heat is a ponderable or an imponderable;* while we recognize by these necessities of thought, that it must be true that all things *either are vertebrates or are not vertebrates*, and *are ponderables or are not ponderables.* So with all negative attributes, of themselves they do not carry with them the necessities of thought; for they imply 'the existence of subjects to which they belong; and this existence, thought of itself cannot affirm,' but can only accept it as a *datum.* It is not allowable accordingly to assume that space and time must, from the necessity of thought, from pure contradictory disjunction, be either finite or infinite (§ 185).

A logical disjunction implies at least two members which are opposed to each other. These

parts thus disjoined are called *Disjunct Members.* When the disjunction lies in the terms, the parts are also called *Divisive Members.*

It implies also that if one member be affirmed the other must be denied. Thus, in the proposition: *A is B or is not B*, if either member be affirmed as true, the other member is not true;—if *A is B*, then it is false that *A is not B.*

It implies, moreover, that the disjunct members are not identical but are different from one another; otherwise the judgment would be no judgment.

If the disjunction be in contrary opposition, as *A is either B or non-B*, the same principles apply; as in *The sponge is animal or vegetable*, *animal* and *vegetable* exclude each other; only one can be true attribute: if either is, the other is not; if either is not, the other is.

Contrary Opposition is of several distinguishable degrees. The highest degree is found in the simple negative form of an attribute: as, *non-B; non-combatant; unjust; inexact; speechless.* A second degree is expressive of mere incompatibility, as *round* and *square*, *Greek* and *Turk.* The same thing cannot be at once round and square, nor the same man at once Greek and Turk. These attributes are in the respective cases incompatible with each other in the same subject. But it does not follow, as is true in contradictory opposition, that the denial of the one of the two implicitly affirms the other: the stick is not

necessarily square if it be not round; it may be of some other shape. So in the higher degree of contrary opposition a man is not necessarily just if he is not unjust.

§ 38. The modifications of the judgment founded on the terms or the matter are threefold, according as they are determined by the Logical Form of the terms, their Quality, or by their Quantity.

By LOGICAL FORM as a principle of division of judgments is meant the thought-form of the terms in distinction from the word-form. This thought-form may be either that which is proper to the concept—the conceptual form—or that which is more proper to the judgment—the assertory form. This distinction gives rise to the two kinds of judgments called the *Categorical* and the *Hypothetical* judgment.

A CATEGORICAL JUDGMENT is one the terms of which have the logical form proper to concepts: as, *Man is mortal; The sun shines.* In this last example, the word-form — *shines* — combines in itself copula and predicate; but it expresses properly the logical or thought-form proper to a predicate; that of an attribute ascribed to the subject, *the sun.*

A HYPOTHETICAL JUDGMENT is one the terms of which are in the form proper to a judgment; as, *If A is B, C is D; If virtue is voluntary, vice is voluntary; That virtue is voluntary implies that vice is voluntary.*

The Hypothetical Judgment has received its name from the language-form in which it more commonly appears. It is ordinarily expressed in the form of an hypothesis or condition indicated by the conjunction *if*. But it must not be supposed that this suppositive or conditional character reaches beyond the terms or affects at all the absolute nature of the attribution. The relation of identity, or of difference, as the case may be, is just as absolute as in any judgment. Through the imperfections of language, the assumed or suppositive character of the *datum* or matter is here signalized by the conjunction, as generally where a judgment is presented as an object of thought we mark it as thus used by a grammatical conjunction, as *if* or *that*.

It may be observed here that as elsewhere more fully exhibited, there are two fundamental relationships in thought essentially determined by quantity, the one between the whole and the part, the other between a part and its complementary part. The first relationship, that between whole and part, is the only one formally exhibited by logicians; and their teachings and illustrations have been confined to that. In their treatment of hypothetical judgments they have experienced the natural evil consequences of partial and one-sided views, their expositions of this product of thought being unsatisfactory and discordant. It is important accordingly to recognize the fact that under the ordinary form of the

Hypothetical judgment, either of the two relationships of quantity, of whole to part and of part to complementary part, may be indicated. Thus in the proposition: *if virtue is free, temperance is free*, the relationship is that of whole to part—*temperance* being one of the *virtues*. In the proposition: *if virtue is free, vice is free*, the relationship is that of part to complementary part, *virtue* and *vice* being complementary parts of the whole *moral action*.

In either case, the antecedent clause is said, indifferently, to *determine*, to *condition*, to *imply*, to *involve*, the consequent clause, or the reverse. For it is to be remarked that the consequent or predicate clause may sometimes condition as well as be conditioned by the antecedent. The different character of the two relationships will naturally control more or less the character of the verbal expression.

§ 39. By the LOGICAL QUALITY OF THE TERMS is intended the specific character of the attribute as causal or substantial, which gives the two species of properties, *actions* and *qualities*. Any object presented to thought in simple apprehension — by perception or intuition — necessarily brings with it the character of activity,—it necessarily impresses and so acts upon the apprehensive sense. As having this character it may be viewed in thought as *a cause* in reference to the result of the action which is in this relation of thought designated *an effect*. The attributes

that are given under this causal view are denominated attributes *of action*, and the judgment as characterized by this causal relation is denominated a *causal judgment*.

But in our thought we may abstract this active property from the object and then it is viewed in thought simply as *a substance* having certain characteristics that are specifically named *qualities*. As cause and effect are necessary correlatives, one ever implying the other, so likewise substance and quality are necessary correlatives. These relations, it is to be observed, are the simple creations of thought. They have no real existence in nature. We can only say of natural realities that they are such that on being presented to thought they admit of being viewed in these relations. What is of special importance to be noticed here is that in the looseness of popular speech which admits of a representation of things in nature under those terms originally proper to thought, it is very easy to slide into the fallacy of attributing to nature the character of necessity which properly belongs only to the relations created by thought. As there is no general thing in nature but all the generality that can be is the product of thought, so there is no purely causal or substantial thing in nature which has its relative effect or relative attribute. These relations with all the characteristics of necessity that attach to them are ideal or logical, not real; they are not seen or heard or discerned by

any sense; while yet, as legitimate creations of thought, they ever must have correspondences in real existence.

Language has furnished appropriate forms for expressing these different species of logical quality in attributes. The attribute in the causal judgment is more properly expressed in the form of the grammatical verb; as, the sun *illuminates* the earth, where the object of the action is expressed and the verb is called a *transitive* verb; the sun *shines*, where the object is not expressed and the verb is called *intransitive*. The copula and the attribute in the causal judgment are generally in language combined, as in *shines*. In the substantial judgment, the attribute is expressed more generally by the grammatical adjective, as, the sun is *bright* (§ 65).

§ 40. By LOGICAL QUANTITY is signified the range of matter comprised in the terms of the judgment.

It is either *Intensive* or *Extensive*.

INTENSIVE QUANTITY, also known as Comprehensive Quantity, is founded in the predicate of the judgment and signifies the range of attributes embraced in it. *Wise*, thus, comprises the attributes of *intelligent* and *end-seeking;* it has a greater intensive quantity than either of these component attributes.

EXTENSIVE QUANTITY is founded in the subject of the judgment and signifies the number of objects embraced in it. *Man*, thus, embracing

European and *American*, is of greater extensive quantity than these latter terms.

Intensive quantity is also denoted by the terms, *comprehension*, *depth*, *content*, and by the preposition *in;* extensive quantity by *extension*, *sphere*, *breadth*, *extent*, and by the preposition *under*.

In each kind of quantity, the most fundamental relationship is of a twofold character: (1) that of the Whole to the Parts; and (2) that of the Parts to one another.

In intensive quantity the whole is a comprehensive whole—a whole of attributes; and the parts are the particular attributes which compose the whole. In extensive quantity, the whole is an extensive whole and is called a *generic* or *universal* whole or a class whole; and its parts are the particular individuals which make up the class.

It is worthy of particular mention here that the second relationship in quantity named above —that of Part to Part—has been to a large extent overlooked in logic, at least relatively to the overriding consideration given to that of Whole and Part. But it is most obvious that the two relationships are co-ordinate, of equal rank, and of like importance in any adequate consideration of the nature and forms of thought. It is in consequence of this oversight in logical science, that that most important movement of thought known as Induction has been so strangely neglected or misunderstood. The Deductive move-

ment of thought is everywhere recognized as resting on the relation of Whole and Part; the Inductive movement as indisputably rests on the co-ordinate relation of Part to Part.

§ 41. In respect to the extensive quantity of the subject judgments have been distributed into *Universal* and *Particular*.

A UNIVERSAL JUDGMENT is one which has for its subject a class embracing all the individuals of the class. The designations of universality are the pronominals *all*, *each*, *every*, *whoever*, *whatever*, *none*, *neither*, and the like. For the most part, however, in common discourse such signs of universality are omitted: as in the proposition *men are mortal*, it is understood that the whole class—*all men*—is meant.

A PARTICULAR JUDGMENT is one which has for its subject a part of a class; as, *some men are wise*. The designations of particularity are, *some*, *many*, *few*, *not all*, and the like.

A judgment which has for its subject a term that does not explicitly designate its quantity has been by some logicians called *indefinite*.

To avoid the ambiguity arising from the uses of this word, Sir William Hamilton suggests the term *preindesignate* to denote propositions in which no mark of quantity is given, and *predesignate* to denote those in which it is stated. *Indefinite*, he thinks, properly applies to particular judgments denoted by such pronominals as *some*, *many*, etc.

A judgment, the subject in which denotes a single object, is sometimes called an *Individual Judgment;* as, *Alexander was a conqueror.*

The exposition of the nature and form of thought in its relations to quantity has been from the time of Aristotle down exceedingly defective and misleading. Aristotle himself expounded thought as it appears in the particular form of the reasoning or syllogism, to which department of the science of thought he gave almost exclusive consideration, and for the most part irrespectively of its quantitative determinations. His most general statements concerning the nature of the proposition or expressed judgment and of the syllogism with its specific forms, do not contain terms denoting quantity. The relations of subject and predicate are expressed by such words as *predicated of*, *belong to* or *inhere in*, and by prepositions. Exemplifications, as perhaps most natural and most intelligible, occur which express relations in quantity; but the fundamental regulative principles of the syllogism avoid them. They are more generic and only embrace them as a generic or class-term may embrace unnamed species. His commentators very naturally explained his doctrine in examples of quantitative relations; and at last came to exhibit all thought as moving in quantity, and even in extensive quantity exclusively. The scholastic logic and its formulastic contrivances were thus fatally defective and erroneous. It would be pre-

mature here to discuss this general subject of quantitative thought. This can properly be done only under the discussion of the concept and of the reasoning, particularly the syllogism, in which the scholastic distinctions of quantity characteristically appear.

II. Extrinsic Modifications of the Judgment.

§ 42. The extrinsic modifications of the judgment are either *mental* or *verbal*. The mental modifications are founded either (1) on the source or ground of the act of thought, giving the *modal* distinctions of judgment — those characterized by logical modality; or (2) on the end or aim of the thought.

§ 43. Logical Modality is that character of thought which respects the judgment as *necessary*, being determined wholly by thought itself, or as *contingent*, being determined more or less by the matter of the thought. As before remarked, all legitimate movements of thought have the character of necessity; they must be so, as otherwise thought would not be thought. But the matter on which thought exerts itself, being presented to it from without the thinking sphere, it may be incompetent for thought to establish as true; it is not necessary, it is contingent. Hence the distinctions in judgments in respect of modality— (1) *Necessary*, also named *Demonstrative* and *Apo-*

dictic; (2) *Contingent*, also named *Problematic* and including *possible* and *probable* and their contraries; and (3) simply *Assertory*, in which the judgment appears as unmodified in this respect.

§ 44. The intelligence in man is but one function of an organic whole; and its activity in producing thought cannot be unaffected by the action of its co-ordinate functions. Thought is colored and shaped by feeling and by purpose; and suffers divers modifications from these. As an activity it involves not only a ground, which, as just noted, gives rise to the distinctions of modality, but equally an end. Thought, from its organic nature, is end-seeking—telic; and this characteristic sometimes becomes prominent so as to determine modifications of the judgment. Language, as the natural embodiment of thought, has marked these modifications of pure thought, by giving to them distinctive forms. The end-seeking, or as they have been called, the *orectic* functions of the human soul, are *desire* and *will* or *purpose*. It has furnished accordingly for judgments as modified by desire what is called the *optative mood* of the verb, and for those modified by will, the *imperative mood*. We have accordingly the distinctions in judgments characterized in reference to the end of the thought: the simply *assertory*, the *optative*, and the *imperative*.

It seems incumbent on logical science to recognize these modifications of thought; while yet it may limit itself mainly to the consideration of

pure thought, or thought as unmodified by the co-ordinate functions of the human soul. Logic has excluded these modified forms of thought from its consideration the more as it has elevated the reasoning to an almost exclusive position in the science, very unnaturally subordinating to it in their treatment the simple judgment itself and the concept, as if the derivation of one truth from another were the comprehensive end of thinking.

§ 45. While to a large extent the different kinds of judgment have, each, their own proper forms of verbal expression, there are certain forms of expression which do not of themselves indicate anything as to the kind of judgment. These different forms of verbal expression so far give rise to certain distinctions in expressed judgments or propositions. Thus the verbal form may be full and complete or it may be more or less elliptical. Propositions in which the verbal statement explicitly contains the three constituents of the judgment—subject, copula, predicate—as, *the sun is bright*, have been called propositions of the *third adjacent;* those in which the copula and the predicate are expressed in one, as, the sun *shines*, are called propositions of the *second adjacent*. It sometimes happens that the entire judgment is expressed in a single word, as Lat. *pluit;* Eng. *it rains*, where *it* is not to be viewed at all as expressing a subject, or as having any meaning other than as a purely rhetorical expletive, used simply like the particle *there*—

there is light—where it simply marks the inversion of the sentence.

Further, the words employed may not of themselves express precisely what is thought. Language is at best but an inadequate embodiment of thought; it addresses the imagination and often only suggests rather than represents the thought. It is right therefore to demand always that what is introduced in thought be as clearly as possible explicated in language. But it would seem as if more were demanded sometimes in enforcing this reasonable postulate than the case allows. Such is the demand of Sir William Hamilton that in an extensive proposition the predicate should, at least in logical science, be overtly quantified, that is, that the range of matter intended should be expressed in words. In an extensive proposition the terms, both subject and predicate, are to be construed as in extensive or numerical quantity. The proposition, thus, *man is rational*, means *all men are some rational beings*, and formal logic, Hamilton insists, should formally recognize all such propositions in their systematic enumeration of propositions. His reasoning seems to be based on the assumption that the relation between subject and predicate is that of a mathematical equation. But this is altogether too narrow a view of thought. This numerically quantitative relation marks only a part of our judgments, which, indeed, for the most part exclude entirely all quantitative relationships

of this kind, except the one comprehensive relation implied in all judgments. As we have seen, judging is attributing—identifying or differencing some attribute in relation to its subject. The judgment, the proposition, wholly omits the specification of the proportion the attribute bears to the subject. The proposition 'man is rational construed as meaning 'all men are rational beings' does not of itself express how great a part men are of rational beings. It does not even express whether the terms may not be perfectly convertible, while still it remains true that the predicate expresses only a part of what might be attributed to the subject—*men*.

In intensive propositions, as, *man is rational*, construed in intensive quantity, the predicate is simply presented as one of the attributes recognized as belonging to the subject; but whether the terms are exactly convertible or not, is not indicated. The judgment must be changed in order to express this convertibility; and then it becomes a different judgment and has its own explication.

The whole discussion of this subject of the quantification of the predicate seems to have proceeded in only a partial view of the nature of thought as attribution, and to have originated at a time when the old views of logic, which looked almost exclusively at thought in the relations of extensive quantity and so classified all propositions as A, E, I, or O, were felt to be defective

and unsound. A great service was rendered to logical science by the "New Analytics," as the system of Hamilton was called. The quantification of the predicate involved the recognition of intensive quantity in thought, co-ordinating it with extensive quantity. The rejection of the entire formulastic logic of the schools as shown by Hamilton to be not only needless and cumbrous, but positively unsound and fallacious, made the further pressing of the quantification of the predicate entirely unnecessary, except for historical and literary uses.

But from the relation of thought to its embodiment in language, which lay at the bottom of this discussion, there is discoverable a ground of distinction in judgments which deserves to be distinctly recognized. If the judgment be presented with a subject and a predicate of equal content, the proposition is called an *Identical proposition;* as, $A = B$; *Columbus was the discoverer of America; Arithmetic is the art of computation.* If the terms are expressed in the same signs or words, the proposition is said to be *Tautological;* as, *A is A; Right is right.*

§ 46. Judgments, it should be added, are characterized in reference to the relationship which they may bear to one another. If any two judgments, thus, have the same matter and form, they are called *Identical*, *Convertible*, *Equipollent.* If of two judgments, one affirms, while the other denies, they are designated as *Conflictive.* If no

opposition exists, they are called in reference to each other *Congruent*.

TABULAR VIEW OF THE MODIFICATIONS OF THE JUDGMENT.

- Intrinsic.
  - Copula.
    - Simple noematic quality.
      - Affirmative.
      - Negative.
    - Disjunctive noematic quality. Disjunctive.
  - Terms.
    - Logical form.
      - Categorical.
      - Hypothetical.
    - Logical quality.
      - Substantial.
      - Causal.
    - Logical quantity.
      - Comprehensive.
      - Extensive.
- Extrinsic.
  - Ground of thought. Modality.
    - Assertory.
    - Apodictic.
    - Problematic.
  - End of thought. Telicity.
    - Interrogative.
    - Optative.
    - Imperative.
  - Language. Verbal form.
    - Full.
    - Elliptical.
    - Identical.

See Appendix, No. II.—Exercises in the kinds of judgment.

CHAPTER V.

CONSTITUENTS OF THE JUDGMENT:—1. THE COPULA.

§ 47. THE copula we have recognized as the more central and vital constituent of the judgment which is ever tri-membral, the more exterior constituents being the subject and the predicate. Aristotle in the First Book of the Prior Analytics, c. xxv., speaks of it under the name of *διάστημα*—the interval, or the part of the judgment standing between the terms, *ὅροι*. Its proper function and import is that of combining the terms so as to give existence to the judgment. It expresses the active life of the thinking subject, which entering into relation with the presented object—the *datum*—analyzes it in its reflective act into the two terms and then creates an organic whole or completed body of thought. But the essence of all thought is attribution, which consists in affirming or denying an attribute of a subject, that is, in identifying or differencing these two terms of the judgment. We have thus the definition of the copula.

THE COPULA is *the attributive or assertive constituent of the judgment.*

It is accordingly the positive or more vital ele-

ment. Its proper verbal expression, *is—is not*, as Aristotle well teaches, insignificant by itself, denotes a union or composition which it is impossible to think without the composites. This uniting or compounding act gives the significance of a living thought to the matter or terms. *Sun*, *bright*, express no thought: *the sun is bright* becomes true, significant, living thought, as the copulative, *is*, unites and organizes the terms otherwise dead to thought.

The copula in thought as the product of the intelligence bears a close analogy to the central and vital principle in the proper product of each of the other functions of the human mind. Thus *the beautiful*, as the product of the imagination or function of form, is ever the expression of some idea in some matter, and the expressing act constitutes the more central and vital element in all beauty—*idea* and *matter* being the other constituents of the tri-membral product. In the same way the product of the will or the function of duty or the good has its central and more vital element in the relation between the subject of duty and its object.

§ 48. The function of the copula is alternatively twofold: it must either affirm or deny—or, in other words, it must identify or difference. This function is proper to thought as the completing act of the intelligence. Perception in the case of a single object necessarily precedes and conditions the reflective intelligence that begins

its work by analyzing the presented object into subject and attribute. The combining of the two through the copula consummates the cognitive act and gives the completed judgment. If it be supposed that the work of comparing subject and predicate often intervenes, this is still to be regarded as only a part of the reflective, judging act which, as a thing of experience, is ever more or less continuous, involving the element of time.

The nature of the relation which the copula thus effects between the terms of the judgment is that of attribution. It has no other conceivable significance. Its function is simply to unite the predicate with the subject, so as to form the organic whole of the judgment; and the nature of this union is that of judging or mentally asserting the relation between them; in other words, of attributing. It is however conscious attribution, an act of the mind in its special function of the intelligence. The activity is limited, it should be noticed, within the sphere of consciousness. The judging act does not reach to the external object presented to the sense; there is no interaction between the reflecting intelligence, between the mind at this cognitive stage, and the outer object, by which that object itself is at all changed in essence or in any real relationship. The sun has not suffered a whit of change by reason of my judging that it is bright. The judgment respects only the object as already apprehended, as already perceived or intuited.

The copula only attributes what in this apprehended object it discerns of attribute or character.

This act of attribution is ever one of identifying or differencing. The discerned attribute or character is recognized as a part of the object—part of what constitutes the object, and so far the same with it. In the case of the primitive judgment, as in our selected type-form of the judgment—*the sun is bright*—which must, if a true judgment, contain the entire essence of a judgment, the object is both single and simple, and the attribute—*bright*—obviously can be none other than the subject, *the sun*. The entire force of the copula is exhausted in recognizing the identity. This view of the copula, as the more vital and characteristic constituent of the judgment, which simple inspection gives us, is confirmed as the true view in all the manifold applications of logical science, in all the manifold developments of scientific knowledge. It is recognized as true in mathematical science, ranked as an exact science, where the copula is expressed in the symbol of the equation, that is, of the quantitative identification. It is implied in the universal sentiment that all proper truth in knowledge lies in the sameness of what is asserted with that of which it is asserted, except in this one respect, that the two are the exact opposites in this relation of attribution—one being what is attributed, the other that to which it is attributed. They are in other words subject and

attribute which, as there is truth, must be the same except in this relation of attribution.

§ 49. The copula, while it is thus the more vital constituent of the judgment, is not the more prominent and obtrusive element. It is in truth exceedingly subtle and slippery in the mental grasp. All thought tends to express itself in words. But language satisfies itself with furnishing the conditions requisite for the communication of thought. It assumes the spontaneous activity of mind, which freely determines itself in whatever direction or course of movement the object presented to it may occasion. These conditions are to a certain extent supplied often by the simple presentation of the object. To point to the sun may suffice to lead another mind to the judgment that the sun exists—is real, is visible. If language be already formed, to utter the word *sun* or the word *bright* may suffice to lead to the judgment that the *sun is bright*, although no assertive element, no copula, is expressed. In the infancy of language this element is not expressed, because not so necessary, and particularly because it is difficult to originate a word by which the idea of assertion shall be so associated by both of two communicating minds as to be an adequate sign of expression for it. Language, moreover, for communication, must necessarily start from some sensible experience common to both communicating minds. All abstract terms are hence derived

from words expressing sensible objects or states. The copula finds its earliest verbal expression in the word expressing existence—*is*. The closest analogue in the outer world to the essential principle of thought is the character of existence or reality. Hence the common and pernicious confounding of the being of thought, which is mere identifying or differencing attribution, with the being of real things. The verb *is*, thus, originally and properly expressing real being or existence, has become the more proper sign or expression of the logical copula. It is often of the highest importance to be mindful of this ambiguous use of this verb, which is both substantive and attributive and expresses both substantial reality and logical identification. The copula, as has been noticed, is often combined, particularly in expressing an attribute of action, with the predicate: as, the sun *is;* the sun *shines;* the sun *gravitates*. In the first of these examples, *is* is equivalent to *is real* or *is existing*. It is erroneous to suppose however that the copula is of itself expressive of reality in the apprehended object. This is a common fallacy even with profound thinkers. As the judgment can never legitimately transcend its *datum*, it can never rightly put into the object any attribute, reality or any other, except as actually presented. It cannot by its act of thinking make real that which is but imaginary or unreal.

Language has supplied a diversity of forms for

expressing divers modifications of the copula or assertive element. Besides the explicit form *is*, and the implicit forms in the grammatical verb generally, it furnishes such forms as, *consists of*, *contains*, *comprehends*, *involves*, and the like.

§ 50. The copula is the element of the judgment to which its characteristics of truth and modality immediately attach. A judgment is true if it affirms of a subject an attribute that belongs to it or denies of it an attribute that does not belong to it; if, in other words, it identifies the same or differences the different. It is false if it asserts the contrary of this—differences the same or identifies the different. Truth and falsehood in their proper and primitive import respect the judgment. The other applications of the terms are derivative.

The characteristic of modality, which points to the ground of the judging act as lying in the realm of thought itself or as being brought to it from without the thought, also attaches to the copula element. Some British logicians, as Whately, Hamilton, and others, loosely embrace among modal judgments such as are modified only in respect to the predicate. Thus the propositions: *Alexander conquered Darius honorably*, *Justice is ever equal*, they classify as modals. The error arises from a defective apprehension of the nature of modality, which they represent as signifying the degree of certainty contained in the judgment. This certainty may depend some-

times on the terms and some relation between them outside of that which is involved in the copula. But in the examples cited *honorably* and *ever* give no significance whatever of degree of certainty in the judgments—the judging acts themselves; they are manifestly modifications of what is predicated. They assert *honorable conquering* and *universal equality* of their respective subjects. The proposition *Alexander possibly conquered Darius*, is of a different character. A contingency attaches to the assertion—the judging act itself. This characteristic, as already shown (§ 43), points to the ground of the judging act, which, inasmuch as it is the product of an activity subject to the conditions of time and hence of causal relationships, may be reasonably looked for in a judgment. The attempt to distinguish between a logical and a psychological judgment is more than futile. It is groundless, and illusive, and mischievous. It looks like an expedient to dodge difficulty or to hide error. As Aristotle held and amply vindicated in his illustrations of the nature and form of thought, judgments are, in respect to the copula itself—in respect to the being or inhering of the predicate in the subject—either simply, or necessarily, or contingently, so. The assertion may be unmodified as to its ground, or inasmuch as this ground may be in the thought or in the matter, may be modified as necessary or as contingent. The distinction is one of great significance in sound logic. It is ob-

viously well grounded. It lies in the nature of thought and respects the copula and the terms of the judgment. It is or should be in legitimate thinking easily discernible. It is helpful to truth. It is properly expressed in language by the modals *necessarily*, *possibly*, *probably*, and the like, and may also be expressed by the inferentials or illatives, *accordingly*, *therefore*, etc.

§ 51. Recent German logic gives but shadowy recognition of the copula as a constituent of the judgment. Prof. Wundt, while expressly stating that "scholastic logic, since Boethius at least, has represented subject, predicate, and copula as essential constituents of the judgment," as expressly pronounces this view to be false, asserting that "the copula is in no way a necessary constituent of the judgment." He is evidently betrayed into error by his too close and unquestioning adherence to grammatical science as the exact correlative of the science of thought. His logic is rather a science of propositions than of thought. Lotze and Ueberweg have little or nothing to say of it as a distinct element.

British logic on the other hand has been harmonious in the recognition of the copula as a distinct and essential element in the judgment.

CHAPTER VI.

CONSTITUENTS OF THE JUDGMENT:—II. THE CONCEPT.

§ 52. THE concept is one of the constituents of the judgment. It appears in each of the terms and is thus of a twofold character. It is subject or it is attribute. The name is applied equally to both—to that of which we think and to that which is thought of something. It is ever to be identified by its being, actually or possibly, explicitly or implicitly, a subject or attribute of a judgment. It is an organic product, and can exist only in and with the judgment as a constituent member. We have seen its genesis as such organic part of an act of thought (§ 30). It can never exist before the judgment, as the limb cannot exist before the living body of which it is a member. It has its appropriate form in the embodiment of thought in language. It is distinguished from a percept or product of the act of perception, by this: that language, as the natural embodiment of thought and of thought only, has no form for a percept any more than it has for a desire or a purpose. The concept stands in a certain vital relation to the percept, as the judgment stands in a close organic, although different

relation to the desire or purpose. The judgment is grounded on the perception and springs from it and so is determined by it; and the concept, as organic part of the judgment, roots itself in the percept as the product of the outer or inner percipient act. The percept obviously can never properly be spoken of as a subject in a judgment. Even the singular subject, as in '*Solomon* was wise,' is a concept for, as organic part of the judgment, it is a true thought-element, a product of the reflective and discursive intelligence.

We have found the entire essence of a judgment and therefore of a concept in the act of thought exerted upon the perceptive or intuitive apprehension of a single and a simple object. But this object thus given in the apprehension as the *datum* for the thought, need not always, as in the type-form example, *the sun is bright*, be presented in this single and simple form. We may have two objects presented to us from entirely different directions and sources, one of which we can make subject and the other the attribute of a judgment. We may have presented to us, thus, the objects *virtue* and *wisdom*. It is a matter of indifference to us here how they were presented to our thought or judgment; how they were apprehended at the time by us; how they became a *datum* to thought. It is enough that they are so. They are, while we apprehend them and compare them, both in our minds, and at the same time; for otherwise all comparison would be out

of the question. One could not come into the mind, and depart when the other entered; they must both be present simultaneously. No matter how quick the one may succeed the other; no matter how often and how rapidly the succession may repeat itself; if the intervals of their presence and absence were measured only by a hair's breadth, even by a mathematical line, there could not by any possibility be any comparison unless literally and truly both were in the mind at one and the same time. The comparison may proceed and continue indefinitely. It may supposably not result in a judgment. But it possibly may; *virtue*, *wisdom*, simultaneously apprehended, may be thought or judged; may be regarded in other words as precisely in the same relation of subject and attribute, as *the sun* and *bright* in our typical examples—may be identified in some respect; and so the judgment may emerge,—*virtue is wisdom*. We have two concepts here distinguished as subject and attribute; the judgment has identified them; has performed precisely the same office as in the typical example. It is true that *virtue* and *wisdom* may have existed in our minds before this comparison and judgment as concepts, having arisen in previous judgments; but this circumstance as to the origin of the *datum* makes no difference as to the essential nature of the judgment as attribution, and therefore so far as terms, the nature of the concepts is in no way different from that of concepts for the first time

emerging in thought. The concept thus ever emerges into life in and with the judgment, whatever may have been the origin or character of the *datum*. It has no existence before the judgment which is the only full, complete act of the intelligence. It exists only as it is conceived, thought, judged; only as a product of the conceiving, thinking, judging intelligence. It is difficult even to conceive of a completed perception of any object except through some character or feature of the object by which it is recognized in itself and distinguished from other objects; and this is at least an implicit attribution or judgment. Certainly the object can receive no name until it has been thus recognized in an attribution and the concept consequently has appeared. It is thus the concept, not the object received, which receives the name. The name is applied to the object as conceived or judged, not as simply perceived.

§ 54. From the genesis of the concept, as has been shown, obviously there must be two fundamentally different kinds of concepts—the subject-concept and the attribute- or predicate-concept. They are congenital; they come to be simultaneously, in and with the judgment of which they are members, as head and trunk in the animal body. They are thus necessary *correlates;* the one necessarily implies the other. Every subject-concept implies a corresponding congenital attribute-concept; and conversely every attri-

bute-concept implies a corresponding subject-concept.

As having thus a common parentage or genesis, immediately in and with the judgment, the two kinds of concept have some properties that are common, and others that are respectively peculiar to the one or the other. They bear also to each other certain important reciprocal relationships.

They alike participate in the specific character of the original *datum*, according as it is assumed as a *cause*, or as a *substance* (§ 39). This specific character is, however, determined rather by the form of the attribute-concept; and will more appropriately receive consideration under the special discussion of that kind of concept. The proper logical quality of the concept will accordingly be exhibited under that head.

Both kinds of concepts are divisible into the two species of *primitive* and *derivative* concepts.

This distinction is one of considerable moment. In linguistic and in metaphysical speculation, grave errors have attended a failure to recognize it, as, for instance, in linguistics, it has been assumed that the primitive word denotes a *general*, or *universal* concept. There is a twofold error here; first, *general* and *universal* denote subject-concepts, not attributes; secondly, a primitive attribute cannot be a composite concept, as all general or universal concepts are.

In the primitive judgment and consequently in

the primitive concept, inasmuch as the object impressing itself on the apprehensive sense in experience must constitute a unit in the consciousness, it must in the judgment be viewed as one. The primitive subject is thus ever *single;* while the primitive attribute is ever *simple.*

The derivation of concepts must be governed by the respective natures or characteristics of the two kinds of concepts.

§ 55. Both kinds of concepts admit alike the two leading distinctions of quantity. Each has however its own specific determinations. The quantity proper to the subject-concept is denominated *extensive* quantity; that proper to the attribute concept, *intensive* or *comprehensive* quantity (§ 40).

As the variations in quantity are twofold, by increase and by decrease, we have two lines of distinctions in quantity in the two respective kinds of concepts · in other words, two lines of derivation from the respective primitives. In both, the process by which a concept is increased is denominated *Amplification;* that by which it is decreased, *Reduction.*

An amplified concept is also called a *composite concept*, and a *synthetic concept.*

It follows from the genesis of all concepts and the relationship thus necessitated between subject-concepts and attribute-concepts, that any variation in quantity effected in one must be under the regulative law of this relationship. Two

subject-concepts arising from differing judgments and consequently having differing correlative attribute concepts cannot be combined into one composite concept; we cannot unite or add two bushels to three dollars, so as to form five bushels or five dollars. They can be united in one only as they may be referred back to two judgments that have the same attribute in each. In like manner no two attributes can logically be united except on condition of having the same logical subject. This is the universal law of logical amplification.

The concept which is thus the necessary condition of the union of two or more concepts is called the *base* of the derived concept.

It may be that in the allowed abbreviations of the process, one generic concept may be added to another, provided the law of the base is observed. But absolutely in all legitimate generic concepts it must be theoretically possible to carry back the formation to a primitive single concept.

§ 56. The amplification of the subject-concept, that is, an increase in extensive quantity, gives as its result the union of two or more subject-concepts into one, which is now called a *generic concept*, or *class concept*. The process is denominated GENERALIZATION. Under the law of logical amplification this can be effected legitimately only on the condition of judgments in which each of the single subjects is connected with the

same attribute that is thus adopted as the base of the derived concept.

The quantity here is extensive, corresponding to numerical quantity in mathematics. However great the increase, that is, whatever be the extent of the amplification, it may always be resolved into primitive concepts which are single or individual. The component concepts may be counted, one by one. The increase may be resolved back into an increase of one by one; an addition of individual to individual.

The process so familiar, that it hardly requires explication, may be exemplified in the formation of the generic concept—*planet*. The primitive judgment is single, as, we will say, *Venus*, which object we recognize through this attribute —*revolving around the sun as the centre of its orbit*. This is the primitive *datum—something revolving, etc.*, given by observation, and single. We name the subject, *Venus*. But a second perception gives us another *thing revolving around the sun as the centre of its orbit;* and we say, *Mercury is a something revolving, etc.* Now we have a common attribute in two different judgments; and on it as a base we unite the two subjects—*Venus* and *Mercury*—and give to the combination, now a class-concept, the name *planet*. We may now ascribe the attribute *revolving, etc.*, to the class—planets, and affirm *the planet to be something revolving, etc.* Extending our observations, and finding the same attribute in divers observed objects, we

obtain similar results in regard to *Mars*, *Jupiter*, *Saturn*, etc. We thus extend our generalization increasing the quantity of the concept *planet*, that is, the number of individuals included in the concept. The only limit to the process is the failure to find another object having the common attribute.

It will occur at once that there is a want of scientific accuracy in this exemplification. All bodies revolving around the sun are not planets. The process as given, however, sufficiently well exemplifies the process in generalizing. It is supposable, further, that this may have been the actual history of the generalization in this case, no other bodies having then been observed which possess the common attribute. It becomes apparent thus that generalizations are not processes going on in nature; there are no such arrangements in the outer world as our genera or classes. The Realist has disappeared from among philosophers; there is no *general man* in actual existence, to which the name *man* exactly corresponds. Further, the generalization, however perfect at the start so far as observation may have extended, is subject to indefinite changes both in content and extent. The common attribute may be found to be too ample or too restricted; it may be incorrectly recognized. The subjects embraced in the generic concept may be multiplied indefinitely in extended observation. Generalization is the imperfect work of

imperfect men and is subject to indefinite correction and change according to the increase of knowledge or the diversification of human needs.

§ 57. There are three noticeable gradations in the process of generalizing which have a definable logical relation to each other and which it is convenient in logical studies to recognize. These gradations are those of *Genus*, *Species*, and *Individual*.

The GENUS is relatively higher, that is, more extensive in its embrace of individual subjects. We speak also of the highest genus; of a higher and a lower genus than the *Species*. The INDIVIDUAL is the lowest in the gradation. SPECIES is intermediate between the highest genus and the individual; it is identical with a genus at any gradation lower than the highest. Any genus lower than the highest may obviously be a species; and any species higher than the lowest may be a genus.

Science in its several departments has recognized intervening gradations varying in number and in name according to the requirements of the particular science. In Natural History are to be found the most numerous gradations. We find distinguished from each other and ranking in the order given, thus, those of *kingdom*, *sub-kingdom*, *class*, *sub-class*, *order*, *tribe*, *family*, *genus*, *sub-genus*, *species*, *sub-species*, *variety* (§ 128). The gradations however are differently ranked and

differently named by different naturalists. The fourfold gradation of *genus*, *species*, *variety*, and *individual*, is convenient and will be found for the most part sufficient for general uses.

§ 58. The amplification of an attribute-concept, that is, the increase of the intensive quantity of a concept, is called DETERMINATION. It is so called because generally the addition of an attribute determines more narrowly the quantity of the correlative subject-concept. Thus by adding the attribute *within the earth's orbit* to the attribute *revolving around the sun*, we reduce the extensive quantity of *planet* to that of *inferior planet*, and attain in our thought a lower species.

The process of determination is as indefinite as to degree as that of generalization. We may descend from the higher class by longer or shorter intervals, attaining gradations of species of indefinitely numerous orders till we reach the individual, which in any order must ever have the largest intensive quantity; it must, in other words, possess the largest number of attributes.

§ 59. The reduction of the extensive quantity of a concept is called DIVISION. We divide, thus, the concept *planet*, considered as a subject-concept, that is, in its extensive quantity, into *inferior* and *superior*. It is effected simply by the process of determination. The movement of thought is from the higher to the lower class.

§ 60. The reduction of the intensive quantity of a concept is called *Partition*. We divide thus

the composite attribute *revolving around the sun within the earth's orbit*, when we separate it into its two component attributes, *revolving around the sun* and *within the earth's orbit*. It is the logical opposite of generalization, as division is the opposite of determination. In Generalization and Division a concept is regarded as a subject-concept; in Determination and Partition, as an attribute-concept. The same name is applied, with perhaps a slight modification as by grammatical inflection, indifferently in each case; so that of itself the word does not indicate the quantity as extensive or intensive; that is, whether it is a proper subject-concept or attribute-concept. The word *man* thus may signify the aggregate of subjects possessing the common attribute of *humanity*, or may be used to signify the aggregate of properties or attributes possessed by *man*. The discrimination in this respect as to quantity must be determined by the particular case. It is by reason of the fundamental character of identity between subject and attribute which is given in the judgment itself that this ambiguity in language arises. And it is because this identification in the judgment is for the most part only partial, affirming sameness only between the subject and a part of its attribute character, that the determinations of classes as to both extensive and intensive quantity are so indefinite and so liable to lead to misconstruction and error.

§ 61. It is obvious that there is a fixed relation between these two kinds of logical quantity, so that as one quantity is increased the other is decreased. Generalization, or increase of extensive quantity, runs parallel with partition or reduction of intensive quantity. Thus, the concept *inferior planet*, having for its attribute *revolving around the sun within the earth's orbit*, is generalized or increased in extension to *planet* by reducing the comprehension to *revolving around the sun*.

But this law, absolute as it is if rightly construed, must not be interpreted to mean that any increase of extension necessarily involves decrease of comprehension; for adding *Uranus* and *Herschel* to the class *planets* involves no reduction of the comprehension—*revolving, etc.* Nor on the other hand does the addition of such attributes as *shining by reflected light*, reduce the extension of the class concept *planet*. The extension can be reduced only by the addition of an attribute not comprehended in the original base; as, *planet* is reduced to *inferior planet* by adding to the base—*revolving around the sun*—a new character—*within the earth's orbit*. So it can be amplified only by the removal of an attribute which is not a part of what is left but a true coordinate with it—as, *man* [= *rational animal*] is amplified to *animal* by removing *rational* from the base, not by removing some constituent of *animal*, as *biped, erect*, or other like *animal* charac-

teristics. The law then must be understood to imply an increase or decrease by the addition or removal of some integral constituent recognized in the amplifying or reducing process.

§ 62. Concepts have been conveniently characterized by certain other relations to one another. Thus in extensive quantity there are the following:

I. The relation of *Exclusion*, when no part of one is in the given case thought as coinciding with any part of the other; as *horse*, *syllogism*.

II. Of *Coextension*, when two concepts contain under them the same subordinate concepts; as *man*, *human*.

III. Of *Subordination*, when one called the *subordinate* is contained within another called in relation to the other the *superordinate;* as *horse*, *animal*.

IV. Of *Co-ordination*, when the two are exclusive of each other but are component or complementary parts of the same whole; as *man*, *horse*, as parts of *animal*.

V. Of *Intersection*, when the content of one is in some part the same as some part of the other; as *stratus* and *nimbus*, a *stratus* cloud sometimes being a *nimbus*.

§ 63. Comprehensive concepts bear to one another the following relations:

I. Of *Agreement*, when they may be connected in thought because actual or conceivable attri-

butes of the same subject; as *bright, round.* They are then said to be *congruent.*

II. Of *Opposition,* when they cannot be so connected in thought, as *virtue, vice; square, round; red, yellow.* Concepts in this relation of opposition are said to be *Conflictive.*

III. Of *Subordination;* as *rational, intellectual,* when one, the *superordinate,* comprehends the other, the *subordinate.*

IV. Of *Co-ordination;* when, while exclusive, they are components of the same whole; as *rational* and *animal* are co-ordinate components of *human.* They are called in this relation *complementaries.*

V. Of *Intersection;* as *just* and *kind* intersect in *courtesy.*

§ 64. Comprehensive concepts or attributes are subdivided in respect to their relation to the subject of which they are predicated into two species: I. *Intrinsic;* II. *Extrinsic.*

Intrinsic attributes are contained within the subject. They are also called *Properties.* They are subdivided into two varieties: (1) *Actions,* when the subject to which they belong is viewed as a cause; as *gravitating, illumining;* and (2) *Qualities,* when the subject is viewed as a substance merely; as *heavy, bright.*

In the case of Intrinsic attributes the subject is viewed as a whole containing parts, which parts are the components of the whole.

Extrinsic attributes are such as characterize the

subject to which they belong in its relations to other objects. They are subdivided into two species: (1) *Conditions*, which stand in the relations of the subject as part to a containing whole; and (2) *Relations Proper*, which lie in the relations of the subject to other co-ordinate parts.

§ 65. The different kinds of concepts are largely but not fully distinguished in language. It is the one function of language to embody in sound the forms of thought in distinctive forms of speech; and the forms and rules of language will be determined directly and chiefly from the forms and relations in the thought to be expressed (§ 204). But human speech is at best but an imperfect work, and has not provided a special word-form for every thought-form. It aims more perfectly than it performs; its ideals are admirable, its realizations sometimes fail. It has given, thus, more or less completely, special forms for the two fundamental classes of concepts; and has also to some degree furnished distinct forms of expression for some of the subordinate species.

The subject-concept has its proper verbal form in the grammatical *noun*, so that we have the definition of a noun as that part of speech which may be used as a subject. But an attribute and also a full judgment may be subjects of which we may think. The attribute, thus treated as a subject, takes, in language, the form of the abstract noun, as *useful* becomes *usefulness*, when

this attribute is made a subject of thought. The judgment in its proper form is often indicated in language to be treated as a subject by the use of certain particles, as *that*, *for*, *if*, etc., or the formal words, *the truth that*, the *proposition that*, and the like: as, *that he has gone is certain; for me to judge would be presumptuous; the truth that to be honest is to be prosperous, is one of general acceptation.*

The attribute-concept, when it signifies an action, is appropriately expressed in language by a verb generally combined with the copula; as, *the sun shines.* When it signifies a simple quality, the appropriate grammatical form is an adjective; as, *the sun is bright* (§ 39).

See Appendix, No. III.—Exercises in Concepts.

CHAPTER VII.

THE CATEGORIES.

§ 66. ALL matter of thought, that is, all thinkables, obviously, must be classed as subjects or as attributes. All subjects may be classed as genera, species, or individuals (§ 57). This classification, it is evident, is the fundamental one of subjects themselves or so far as mere subjects, and may be very helpful to thought and knowledge, if it be not rather ranked as a necessity. Certainly science could make little progress without generalization. Science has indeed often but inadequately been defined as generalized knowledge. A classification of attributes, fundamental and comprehensive, may be presumed to be alike serviceable to thought. Such a classification, like that of subjects into genera, species, and individuals, must be made in the realm of pure thought alone and irrespectively of the *datum* to thought. This *datum*, indeed, in the primitive act of thought, is ever both individual and simple. The world around us does not present to us a generic thing, or a specific thing, but only an individual. It presents to us only the simple characteristic. It is the function of thought afterwards to analyze it into constituent characters or to amplify it

by uniting it with those presented in other *data*. The most fundamental classification of attributes is accordingly that of the attributes of pure thought. This order of attributes is distinguishable from two other orders to be noticed hereafter; the three orders of categories being (1) the generic attributes of pure thought; (2) those of thing—the real; and (3) those of thought-thing —cause and substance (§§ 191, 194).

The Greek for attribute or predicate was category—*κατηγορία*. Categories of pure thought are accordingly attributes or predicates that are discernible in thought itself irrespectively of its *datum*, and the categories of pure thought are the generic attributes—the highest classes of attributes of pure thought. They are obtained just as the attributes belonging to any other object, the attributes of *the sun*, for example, by simple inspection. They are in no proper sense innate. They are not *apriori* to all thought, any more than *brightness* is *apriori* to the attribution of this character to *the sun*. It is a fundamental fallacy in metaphysical speculation to assume them. They arise in the mind simply by observation—by inner perception or intuition. A thought is present to the mind, and certain characters or properties are discerned in it. They have their rise thus in experience, are founded on actual thought as a matter of experience. They are improperly denominated *first truths* or *first principles*, in the sense of being absolutely first,

that is, before all experience, before all thought, since they first emerge in an act of thought. They are by no means precedent conditions of thought, any more than *brightness* is a precedent condition of the *sun*.

As the attributes of pure thought, the categories are the attributes of all thought, and may be discerned in any particular instance of thought. They are all discernible, in fact, in the judgment which, as already shown, is the proper form of the intelligence in a full completed stage of its action. But the essential character of the judgment is attribution, the conscious connection of an attribute with a subject in identification or differencing. A fundamental character of all thought is accordingly this of attributiveness. Where there is attribution there is thought; where there is no attribution there is no thought. This general character of thought is specified into identification and differencing. But the category is named from the positive form—*identity*.

This category of identity is the most fundamental and most characteristic of all attributes. It is implied in every other. It is the principle of all thought; the recognized principle of all science.

§ 68. The category of identity is discerned in the judgment taken as a whole. There are two other categories of pure thought which are founded respectively on the organic constituents of the judgment—the copula, and the terms.

The copula in itself as mere copula we have found (§§ 43, 50) to be modified in reference to its ground, as necessary or contingent, having its ground in thought itself or in what is extrinsic to thought. All thought, accordingly, may be characterized as *necessary* or *contingent*. We have thus the character of modality attaching to all thought: or, as before, naming the category from its higher form, the category of *necessity*. The proposition, or the judgment expressed in language, may appear as modified or not in this respect. We have thus the three classes of propositions, the necessary, the contingent, and the simply assertory or unmodified judgment (§ 44).

§ 69. Looking once more at the other organic constituent of the primitive judgment, *the terms*, we observe subject placed over against attribute —one as single over against one as simple. One single and one simple make the two terms. *One* and *one* are thus given to us as characteristic of every thought. This character is what is named *quantity*. One and one are two. The character is displayed as it appears in the amplification of the subject-concept. Its modifications are by increase or decrease, which is in the last analysis increase or decrease by ones or single units. This is one of the two species of quantity, *numerical*, founded immediately on the subject-concept. The other species, *spacial* quantity, is founded immediately on the attribute-concept. Its amplification is effected by analysis, as subject-amplifi-

cation is by synthesis. But the amplification is strictly quantitative as one constituent character or attribute after another is attained by the discerning analysis.

The third category of pure thought thus is *quantity*. All thought is in respect to its terminal constituency, quantitative. And it is of two species, numerical and spacial, as determined by the specific difference of subject and attribute-concept. Subordinate characterizations from a diversity of principles will come under each of these categories of quantity.

The most important character belonging to these fundamental categories of pure thought is, that they are the universally necessary attributes of all legitimate thought. They are independent of the *datum*, the object of thought given from without. They inhere in the very nature of thought, while in no sense *apriori* to thought; they are given in the discerned nature of thought and enter into its very essence.

It will be observed that these three categories are exhaustive of this order of attributes—as founded on the trimembral constitution of thought.

CHAPTER VIII.

THE REASONING.

§ 70. THE REASONING *is a movement of thought from one judgment to another.*

It ever begins with a judgment, and moving from it as a starting-point ends in another form of judgment. It has two parts: the *Antecedent*, which is the *datum*, and consists of the judgment or judgments from which the movement proceeds, otherwise called the *premise* or *premises;* and the *Consequent*, which is the result of the movement and is otherwise called the *Conclusion.*

The Reasoning, as compared with the judgment, is a composite of two or more complete units of thought, the simple judgment being, as has been seen, a proper organic unit, although trimembral—a continuous movement that can be marked off into stages but that never breaks its continuity.

It must be recognized as a legitimate and valid movement of thought if it move within the limits of the regulative laws of thought and in its legitimate forms. For thought, as a product of an activity, is in its very nature continuous ; and nothing in its nature limits its continuousness. The simple judgment is continuous as it moves

from the apprehended *dàtum* to the consummated thought uniting or separating subject and attribute, which themselves are attained, in all primitive thought at least, only by an inchoative analytic process. So thought loses nothing of its own proper purity and soundness in continuing from one consummated movement to another. A legitimate reasoning accordingly gives as its result a necessary truth, so far as it depends on the movement itself. The *datum* may be false, or only problematically true; and then the conclusion of the reasoning movement has no guarantee of its truth. Given a true antecedent and a legitimate reasoning, and the conclusion must participate in the necessities of absolute truth.

The proper signs in language of a reasoning are the so-called illatives, such as *therefore*, *consequently*, *since*, *because*, *for*, *inasmuch as*, etc.

§ 71. If the movement of thought in a reasoning be by only a single stage, it is called an *Immediate Reasoning;* if by two or more stages, a *Mediate Reasoning*. In this latter case the movement is mediated by one or more judgments.

IMMEDIATE REASONINGS.

§ 72. Immediate Reasonings are of several kinds.

1. One kind of immediate reasonings is called *Conversion*. It consists in a simple transposition of the terms—the subject and the predicate—in

the antecedent or given judgment: as *A is B; therefore, B is A.* The resulting judgment is called, in relation to the antecedent, the *converse.* Thus *B is A* is the converse of *A is B.*

It will at once occur that conversion or transposition of the subject and predicate cannot be good in all cases, certainly unless the thought be more restricted than the ordinary uses of language indicate; as, from the antecedent *All men are animals,* we cannot move legitimately to the converse, *all animals are men.* The trouble arises from the inadequacy of the verbal expression which does not always exactly embody the thought (§ 45). If we interpret accurately the proposition we find the meaning to be simply that *men are a part of the class animals.* With this interpretation the conversion is valid: *a part of the class animals are men.* So in the intensive proposition, *man is animal,* we mean simply that *man is,* as to one attribute, *animal.* This proposition we may convert: *animal is one attribute of man*—one part of the intensive concept *man.* Every true proposition truly interpreted, every true judgment, is, in the positive or affirmative form, a simple identification of the contents of two concepts; one is identical with the other; and the converse proposition, $b = a$, must be as true as $a = b$. Logical conversion has accordingly to do not so much with judgments as with propositions, or the verbal statements of judgments. In order to be legitimated, therefore, some

change in the verbal expression, in order that it may be exactly adequate to the judgment or thought, may often be necessary.

If no verbal change is required, the conversion is said to be *simple*. *No A is B;* therefore conversely, *No B is A*, is an example of simple conversion. When some verbal change is required, it has been called *conversio per accidens:* as, *all men are mortal;* conversely, *some mortals are men.*

The ruling principle of method in logical conversion will respect the verbal expression of the quantity in each of the terms (§ 143).

A species of logical conversion, called *Contraposition*, in which the quality of the copula is transferred to one of the terms, has been recognized by logicians: as, *No A is B;* therefore, conversely, *no non-B is A*. It proceeds from a fundamental error in assuming that logical quality is thus universally transferable. (See § 145.)

§ 73. II. A second kind of immediate reasonings is *Restriction*. It is of two species: *Modal Restriction*, which respects the copula; and *Quantitative Restriction*, which respects the terms of the antecedent.

MODAL RESTRICTION reduces the modality of the antecedent; as, *A must be B, therefore A is B; A is B, therefore A is possibly B.*

The gradations of modality have been given in § 43. Each higher gradation contains the lower. The highest, the necessary in thought, must con-

tain the simply true; the true must contain the probable, and the probable, the possible.

§ 74. QUANTITATIVE RESTRICTION reduces the quantity of the terms of the antecedent. Thus from the proposition: *Planets are heavenly bodies*, we may infer, by reducing the subject, that *the inferior planets are heavenly bodies—Mercury is a heavenly body*—and, by reducing the predicate, that *planets are bodies.*

The validity of this movement of thought is evident from the nature of logical quantity (§ 40; c. vi.).

§ 75. A third kind of immediate reasonings is DISJUNCTION, which consists in deriving from a disjunctive judgment a conclusion in regard to its members; as *A is either B or C; therefore, B is not C; The sponge is animal or vegetable; it is not vegetable, therefore it is animal.*

This movement is validated by the nature of logical disjunction (§ 36).

§ 76. A fourth kind of immediate reasonings is TRANSFERENCE, in which a derivation is made of quality or of modality, from the copula of the antecedent to the terms, or, conversely, from the matter to the copula.

Transference of Quality may be made in two ways: (1) from affirmation to negation of opposite; as, *A is B; ∴ A is not non-B; The frog is vertebrate; therefore, it is not invertebrate;* also, *A is non-B; ∴ A is not B; The oyster is invertebrate; therefore, it is not vertebrate;* (2) from

negation to affirmation of opposite; as, *A is not B; ∴ A is non-B; The oyster is not a vertebrate; therefore, it is an invertebrate;* also, *A is not non-B; ∴ A is B; the frog is not invertebrate; therefore, it is vertebrate.* The movement in the first way is valid; in the second, not always; as we cannot infer from the proposition: *The crystal is not a vertebrate;* that, *therefore, it is an invertebrate* (§ 37).

Transference of modality from the matter to the copula is always valid, as, *The square root of x^2 is a possible negative; therefore, the square root of x^2 is possibly negative.* But the converse transference from the copula to the matter is not necessarily valid; as, it is conceivable at least that *the square root of $-x^2$ is possibly negative,* for the disjunctive nature of thought allows us to think that it is or that it is not, while it cannot be true that *the square root of $-x^2$ is a possible negative.*

§ 77. A fifth kind of immediate reasonings is COMPOSITION, in which the subjects of several judgments having the same predicate, or the predicates of judgments having the same subject, are united. This movement of thought is what has already been described as amplification of concepts (§ 55).

See Appendix, No. IV.—Exercises in Immediate Reasonings.

II. Mediate Reasonings.

§ 78. Mediate reasonings, by distinction called *Syllogisms*, are derivations of judgments through other judgments.

Syllogisms are of two general classes: 1. *Categorical*, in which the derivation is through the matter of the judgment, consequently through a concept; 2. *Conditional*, in which the derivation is through the copula.

§ 79. All categorical reasoning is in the one or the other of the two general modes, Deduction and Induction.

In Deduction, the movement of thought in the reasoning is ever from a whole to a part. It is directly founded on the principle of the concept, that the attribute belonging to any class-concept belongs to each individual composing the class; or, in comprehensive quantity, that any component part of a composite attribute belongs to the subject to which the composite attribute belongs.

The fundamental law of *Deduction* accordingly is: every concept may be identified with any of its parts as such part; at the same time it is differenced from each part, as whole from a part.

There are thus to be recognized the positive and the prohibitory sides of deductive law. The relation on which the law and the derivation in thought are founded is that of whole to part, and this relation presents the two phases—the whole

is partially the same as its part, and the whole is not wholly the same as its part, that is, it is different from it. Thus if we have a class-concept, *man*, made up of the single concepts *John*, *James*, *Peter*, on account of each of these individuals having the common attribute *rational;* then *man* is *partially* the same as *rational.* But as any given attribute is but a part of its subject, we are prohibited from ever using language importing that it is altogether the same as its subject—that the part is the same as the whole,—that *rational* is *man* in the sense that *all rationality is human.* The doctrine of method as applied to deductive reasoning accordingly recognizes these two phases of the deductive law.

§ 80. In Induction, the movement of thought in the reasoning is from one part to another part of the same whole. It is also founded on the principle of the concept that each individual of the class-concept has the same attribute; or, in comprehensive quantity, that each of the component attributes belongs to the same subject. It follows that if there be found individuals belonging to the same class, the common attribute of the class found in any individual must belong to any other individual of the same class. Thus the zoologist forms a class of animals having the common attribute of *chewing the cud.* He discovers that an individual of this class is *herbivorous* and that this is not an attribute that is accidental to the individual that he observes

but pertaining to it as *ruminant*. From this observation that one *ruminant* is *herbivorous*, he induces that any other is *herbivorous*. He induces accordingly from the recognition of existing species of ruminants being herbivorous that every extinct species was also herbivorous. So the botanist, finding a specimen of a new flower to be *pentandrous*, having five stamens, infers that every other of the same species is also pentandrous. In like manner in comprehensive quantity if there be found any one of the attributes making up a composite attribute belonging to a subject, then to that subject must belong any other of the component attributes. Thus if from the judgments, *man is intelligent*, *man is susceptible*, *man is free*, we gather up these divers attributes *intelligent*, *susceptible*, *free*, into one composite attribute *rational*, then if we find in *man* as *rational* the attribute of *intelligent*, we conclude from his possessing that and being *rational*, he must be *free*,—we infer the attribute of *freedom* from the attribute of *intelligence* as parts of the same composite attribute *rational* belonging to *man*. If the concept *rational* has been legitimately made up, then it is clear that, in this induction from one component attribute to another, we are but retracing legitimate steps of thought.

But it is to be observed that the whole which is thus reached by the process of logical composition combined with induction is ever to be

taken as a distributive whole. We may thus induce from observing one of the seed-cells of an orange to be wedge-shaped that every other seed-cell of the orange is wedge-shaped, and so by logical composition conclude that all the cells are wedge-shaped. But we cannot conclude that the orange as a single and simple whole, thus constituted of seed-cells, is wedge-shaped. Induction is thus characteristically and purely a movement of thought that leads from one part to a like part of the same whole, and ever of itself concludes in a part, never in a whole as such. It is carefully to be discriminated from Generalization, with which it has been confounded even by logicians (§ 164).

The distinction which some logicians have drawn between *formal* and *real* induction as essential, does not exist any more than such a distinction between *formal* and *real* arithmetic. As addition and subtraction are the same essentially in the pure form of numbers as when applied to measuring acres or bushels, so, formal and real induction are the same essentially, the one being pure form, the other mere application to existing things.

§ 81. Categorical syllogisms are subdivided into two classes, Deductive and Inductive.

A DEDUCTIVE SYLLOGISM is a mediate reasoning in which the derivation is from a whole to a part through a concept which is respectively a part of that whole and a whole of that part; as,

all men are mortal; the Grecian heroes were men; therefore, the Grecian heroes were mortal.

In this example the reasoning turns on the concept *men* in the second proposition; only as this concept is recognized as part of the concept *all men*, and as the concept *Grecian heroes* is recognized as part of the concept *men*, can the reasoning be accepted as valid. It is obvious that on the principle of the concept the reasoning is legitimate; for every attribute which belongs to a class must belong to every part of the class. If consequently we have recognized an attribute as belonging to a class, and have also recognized one or more objects as belonging to any part of that class, we must recognize the attribute as belonging to each or any of these objects.

The one principle accordingly on which the syllogism depends is this: that *a part of a part is a part of the whole*, so that what is true of the whole is true of every part. If we represent the larger whole by W, the lower part by P, and the intermediate concept by M, the formula of the deductive syllogism will be, *M is W; P is M; therefore P is W.*

The principle applies as well to comprehensive as to extensive quantity. If thus the attribute *rational* is part of the attribute *human*, and the attribute *intelligent* is part of the attribute *rational*, *intelligent* is part of *human;* so that of

whatever subject *human* may be predicated, *intelligent* may also be predicated.

§ 82. In every valid deductive syllogism there must be three terms and there can be only three terms. They are called respectively the *major*, the *middle*, and the *minor* terms.

The minor term moreover must be a part of the middle, and the middle a like larger part of the major term.

§ 83. In every single deductive syllogism there must be three and there can be only three judgments expressed in three propositions. They are called respectively the *major premise*, also the *sumption;* the *minor premise*, also the *subsumption;* and the *conclusion.*

THE MAJOR PREMISE OR SUMPTION has for its terms the major and the middle terms; as, in the example given, the major term is *mortal*, and the middle term is *men.*

THE MINOR PREMISE OR SUBSUMPTION has for its terms the minor and the middle, as, in the example given, the minor term *Grecian heroes* and the middle term *men.*

THE CONCLUSION has for its terms the minor and the major terms; as *Grecian heroes* and *mortal.*

The major premise and the minor premise together make up the *antecedent;* and the consequent of a syllogism is called the *conclusion.*

§ 84. The order in which the judgments are expressed is of no importance to the validity of the

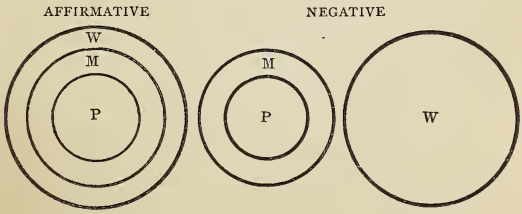
reasoning. We may reason equally well: *The Grecian heroes were men; all men are mortal; therefore, the Grecian heroes were mortal;* or *the Grecian heroes were mortal; for they were men;* and *all men are mortal.* The essential elements of every valid deductive syllogism are three terms, one of which, the major, contains the second—the *middle*—which itself contains the third—the *minor;* and these three terms must be framed in three judgments, one of which, the major premise or the sumption, must identify or difference the middle and the major terms; the second of which, the minor premise or the subsumption, must identify the minor and the middle terms; and the third or the conclusion must identify or difference the minor and the major terms.

It is not necessary that all of the judgments should be expressed. Either of the premises may be implied; as *the Grecian heroes were men; therefore they were mortal;* or *all men are mortal; therefore the Grecian heroes were mortal.* A syllogism in which thus one of the premises is suppressed is called an *enthymeme.*

§ 85. The major premise of a deductive syllogism may identify or difference; that is, may be either an affirmative or a negative judgment. The minor premise must always be affirmative; for if the minor term is not a part of the middle term, there is no connection shown of it with the major term. Thus, from the judgments *Man is mortal; Gabriel is not a man,* there could be no

conclusion, because the minor term *Gabriel* is not recognized as a part of the middle term, *man*. The conclusion identifies or differences, that is, is affirmative or negative, according as the major premise is affirmative or negative. Thus we have the negative form: *man is not omniscient; Newton was a man; therefore he was not omniscient.* Here the major premise excludes the class man from the larger class *omniscient beings* or *things;* the minor premise includes the minor term *Newton* in the class *man;* and therefore *Newton* as a part of this class must be excluded from the attribute.

The relationship between the three terms of a syllogism is well symbolized by three circles; the smallest circle symbolizing the minor term contained in another symbolizing the middle term; and this either contained in the third as when the major premise is affirmative or excluded from it as when that premise is negative. Denoting the terms by letters, the major by W (whole), the middle by M, and the minor by P (part), we have thus:—



§ 86. Logicians formerly were accustomed to distribute syllogisms into classes in reference to the quality and quantity of the judgments, and also in reference to the position of the middle term in the statement of the syllogism. But the more recent logic which has recognized disjunctive quality as well as that of affirmation and negation in judgments, and also comprehensive quantity as a coördinate with extensive quantity, and has, moreover, regarded the form of statement as immaterial to the integrity and validity of a reasoning, has discarded these distinctions as not well-founded, and therefore as fallacious. As however these distinctions enter so largely into logical literature it may be of service to know what they were, while still regarded as untrustworthy.

Logical Mood is the distinction of syllogisms in reference to the quality and quantity of the propositions. There were recognized four kinds of propositions distinguished in reference to their quality and quantity—four moods:—

1. *Universal Affirmative*, as *all men are mortal*, designated by the letter A;

2. *Universal Negative*, as *no man is omniscient*, designated by E;

3. *Particular Affirmative*, as *some men are wise*, designated by I; and

4. *Particular Negative*, as *some men are not wise*, designated by O.

The possible combinations of these four propo-

sitions, A, E, I, O, in the three propositions of a syllogism are sixty-four (4 x 4 x 4); but only eleven of these are valid, each of the other fifty-three wanting some element essential to a sound reasoning.

It has been suggested that the memory may be aided in retaining these distinctions by bearing in mind that the two letters denoting affirmative judgments—*A* and *I*—appear in the Latin *affirmo*, I affirm, and those denoting negative judgments appear in *nego*, I deny.

LOGICAL FIGURE is the distinction of syllogisms in reference to the position of the middle term. There are in the system attributed to Galen (§ 5) four figures or different positions which the middle term may occupy :—

1. *The first figure*, in which the middle term is the subject of the major and the predicate of the minor premise, as *man is mortal; Caius is a man; therefore Caius is mortal.*

2. *The second figure*, in which the middle term is predicate in each premise, as *ruminant animals are not carnivorous; the lion is carnivorous; therefore the lion is not a ruminant.*

3. *The third figure*, in which the middle term is subject in each premise; as *whales are warm-blooded animals; whales are aquatic; therefore some aquatic animals are warm-blooded.*

4. *The fourth figure*, in which the middle term is predicate of the major and subject of the minor, as *all greyhounds are dogs; all dogs are*

quadrupeds; therefore some quadrupeds are greyhounds.

Of the valid moods there are four in each of the first and second figures; six in the third; and five in the fourth, making nineteen in all. As the first was esteemed the only perfect figure, a very ingenious system of rules was devised for the reduction of syllogisms in the three lower figures to the first. They are indicated in the following hexameter verses in barbarous Latin:—

Fig. 1. *Barbara, Celarent, Darii, Ferioque*, prioris;
Fig. 2. *Cesare, Camestres, Festino, Baroko*, secundae;
Fig. 3. Tertia, *Darapti, Disamis, Datisi, Felapton, Bokardo, Ferison*, habet; Quarta insuper habet.
Fig. 4. *Bramantip, Camenes, Dimaris, Fesapo, Fresison.*

Slight variations from this more current form occur. For the interpretation of these verses it may be said that the figures are the three Aristotelian with the one attributed to Galen added, making four in all. The moods, which include all that are valid, are represented in the italicized words, *Barbara*, etc., by the vowels, *a, c, i, o;* as, in *Barbara*, the three *a's* represent that the syllogism is constituted of three universal affirmative propositions. The initial consonants, *b, c, d, f*, indicate the moods of the first figure to which the several moods in the other figures are to be reduced, as *Camenes* is to be reduced to *Celarent*. The other significant consonants—*m, s, p*, and *k*—indicate the particular process of reduc-

tion, as *m*, by transposition of the premises; *s*, by simple conversion; *p*, by conversion *per accidens;* and *k* signifies reduction *ad impossible*. The other consonants—*r*, *n*, and *t*—are insignificant.

But Sir William Hamilton has shown that these rules are unphilosophical and fallacious. They are unnecessary and cumbrous, as the simple application of the one principle in all deductive syllogisms is more easy and is ever conclusive and valid. This principle is, that whatever attribute is affirmed or denied of a whole, may be affirmed or denied of any part of that whole. The sole function of the mediating judgment, which is the minor premise, is to identify a lower part expressed in the minor term as part of a higher expressed in the middle term.

See Appendix, No. VI.—Exercises in Deductive Reasonings.

§ 87. An Inductive Syllogism is a mediate reasoning in which the derivation is from one part to another complementary part of the same whole; as *Venus reflects the light of the sun; Mercury is a part of the same class of bodies as Venus; therefore, Mercury reflects the light of the sun; P is W; C is complementary of P; therefore C is W.*

The derivation is mediated here through the middle term *Venus* (P) which in the second judgment of the syllogism is identified as a like part

of the same class—*Mercury* (C)—as its complementary.

The validity of this movement of thought is obvious, like that of the deductive syllogism, from the very nature of the concept. Each kind of reasoning implies one higher concept which is a synthesis of other lower concepts and ultimately of single or simple concepts, and also implies two other of such lower concepts which are consequently parts of this higher. In a deductive syllogism these parts are unlike parts, the one being a whole to the other and containing it; in an inductive syllogism the two are like parts, that is, are complementary each of the other.

We should indeed beforehand presume from the very nature of a synthetic concept, that there would be two entirely distinct derivations of thought; one being a movement of thought between a whole and a part, and the other, a movement of thought between the parts that make up the synthetic concept. The one is as natural and legitimate as the other. Both are alike serviceable in the advancement of knowledge. Similar objections have been urged against each with like failure, and similar difficulties have arisen in the application of the pure logical principles of the two processes to the knowledge of real objects. It may be added that probably because Aristotle, the father of logical science, omitted to elaborate a science of induction as he did a science of deduction, logicians have for the most

part either passed over induction entirely or treated it in the most meager and unsatisfactory manner.

Aristotle distinctly recognizes the inductive movement. He presents it as the opposite of deduction, and although he did not elaborate his treatment of it as he did that of deduction, so far at least as we know, he yet sufficiently indicates its nature. In Book II. of the Prior Analytics, he says of it that it "connects in thought either of two terms with the middle." His exemplification is this: Let A be *longlived;* B, the middle, *bileless*; C, *man*, *horse*, or *mule*—the individual having those attributes. If now *man*, C, is *longlived* and is also *bileless*, and, he adds, if *man* is convertible with *bileless* and does not transcend the middle, then the *bileless* is *longlived—B is A*. Whatever animal is found to be *bileless*, it may accordingly be induced, will be found to be *longlived*. The reasoning he validates by a principle he had previously demonstrated, that if a subject has two attributes, and is found any where with either, the other may be inferred. His example as given above is a good example of induction in which one attribute is induced from another through the whole to which they both belong.

Induction moves alike in extensive and in comprehensive quantity;—in other words we may induce alike to subjects or to attributes. Thus if *Greek* and *Roman* are recognized as comple-

mentaries in the class of *classic authors;* then if the Greek Homer is a classic poet, the Roman Virgil is also. So if swans and ducks are complementary of each other; then if the duck is web-footed, the swan is also. In the same way recognizing the antelope as complementary, that is, as belonging to the same class as the ox, we induce from the fact that the ox chews the cud that the antelope does also; and from the fact that the ox has a divided hoof that the antelope has also.

The wide range of this process in the extension of Natural Science is seen at once from the fact that any attribute whatever found to belong to one of a given class so far as belonging to the class may be induced to belong to every other individual of the class. Thus all observations on the respiration, circulation of the blood, digestion, muscular contraction, and the like in brute animals of any class are grounds of induction, that the same things are true of man also so far as animal. And on the other hand, finding two flowers to have the same characters, if one belongs to a known class, we do not hesitate to place the other in the same class. In the former case we induce from a class-concept as base to attribute; in the latter from an attribute concept as base to individual or generic subject.

§ 88. In the inductive syllogism there are necessarily three terms: a major, a minor, and a middle; which last is the complementary of the

minor and consequently a like part of the major term.

These three terms appear, in the full statement of the syllogism, in three propositions. One of these, the major premise or the sumption, identifies or differences the middle and the major terms; the minor premise or the subsumption identifies the middle with the minor as its complementary; and the conclusion identifies or differences the minor and the major terms.

Thus, as in the deductive syllogism, the sumption may be either affirmative or negative; the subsumption must be affirmative, for only as the minor and middle terms are recognized as complementaries can there be any movement of thought; the conclusion is affirmative or negative, according as the sumption is affirmative or negative.

It will be observed that the *middle term* which in the deductive syllogism is both a middle in quantity between the major and minor terms, and also a middle as means of mediation between the two, is in the exposition of the inductive syllogism not a middle in quantity, but only a middle as means of mediation;—it is a like complementary part, implying another like part and a whole to which both parts belong.

See Appendix, No. VII.—Exercises in Inductive Reasonings.

§ 89. The second class of Mediate Reasonings is the *Conditional* which mediates the derivation

of the thought through a judgment; as *If virtue is voluntary, vice is voluntary; but virtue is voluntary; therefore, vice is voluntary; If A is B, C is D; but C is not D; therefore A is not B.*

The reasoning here, it will be observed, turns wholly upon the copula (*is, is not*),—upon the affirmation or the negation, that is, upon the essence of the judgment. In this respect conditional syllogisms differ from categorical syllogisms in which the reasoning turns upon the matter of the thought, not on the copula.

§ 90. A conditional syllogism may proceed in either of two modes:—

1. *The Affirmative, the Ponent, or the Constructive mode*, as *If A is B, C is D; but A is B; therefore C is D.*

2. *The Negative, the Tollent, or the Destructive mode;* as *If A is B, C is D; but C is not D; therefore A is not B.*

The judgment on which the reasoning turns is termed the *conditioning* judgment. In the last example, the conditioning judgment is *but C is not D.*

§ 91. Conditional Syllogisms are distinguished, in respect of Quality, into two kinds—*Hypothetical*, proceeding in Simple Quality; and *Disjunctive*, proceeding in Disjunctive Quality.

§ 92. A HYPOTHETICAL SYLLOGISM is a mediate conditional reasoning in which a new judgment is derived from a hypothetical judgment

through the matter, which is here ever a judgment; as *If A is, B is; but A is; therefore B is.*

As shown in the exposition of the Hypothetical judgment (§ 38) the meaning of the first proposition in this syllogism is simply this: the judgment that *A is* includes the truth or judgment that *B is*. The reasoning thus explicated is seen to move in simple quality, as it simply identifies the judgment that *B is* with the judgment *A is*, as one of its parts, and is thus distinguished from the disjunctive syllogism. But this judgment on which the reasoning turns is a virtual term—a part of the matter; it is a judgment used as a subject and it is upon the fact that this subject is a judgment that the reasoning turns. It is in this distinguished from the Categorical Syllogism in which the reasoning turns not upon a judgment but upon a concept.

§ 93. The Hypothetical Syllogism consists of three propositions—a sumption which is a hypothetical proposition having judgments not concepts for its matter, either of which may be taken as the conditioning judgment; a subsumption which affirms the antecedent or denies the consequent of this hypothetical sumption; and a conclusion which affirms the consequent or denies the antecedent of the sumption, according as the subsumption affirms or denies.

§ 94. It may proceed either in the Affirmative, otherwise called the Ponent, also Constructive, mode; or in the Negative, Tollent, or Destruc-

tive mode. Thus having the sumption *If A is, B is;* in the Ponent mode, we affirm in the subsumption the antecedent *But A is;* and then in the conclusion, affirm the consequent *Therefore B is.*

In the Tollent mode, we deny the consequent in the subsumption, *But B is not;* and in the conclusion deny the antecedent *Therefore A is not.* For an example in concrete matter, we may take the sumption, *If matter is inert, there must be mind to move it.* In the Ponent mode, the reasoning would run: Subsumption, *But matter is inert;* Conclusion, *Therefore there must be mind to move it.*

In the Tollent mode it would proceed: Subsumption, *But there is no mind to move it;* Conclusion, *Therefore matter is not inert.*

If it be asked why there might not be two other modes of proceeding in this syllogism, one by denying the antecedent in the subsumption and then in the conclusion denying the consequent of the sumption, as *If A is, B is; but A is not, therefore B is not;* the other by affirming the consequent and so affirming the antecedent, as *If A is, B is; but B is, therefore A is;* the answer is that the *datum* itself makes no provision for such a movement; it says nothing in the case that *A is not;* but only supposes that *A is;* and this, it affirms, involves the consequent. So it makes affirmation as to what will be if *B is* only so far as the judgment that *A is* involves it. It may

be that it will be true that *B is* for some other reason than that *A is*, so that if *A is not*, still it may be that *B is;* as in the sumption, *if matter is inert, there must be mind to move it*, if the consequent were denied, it would not follow either that matter were inert or were not. In other words, in every hypothetical sumption, the antecedent involves the consequent, not the consequent the antecedent.

§ 95. A variety of the hypothetical reasoning having for its sumption a hypothetico-disjunctive judgment or a dilemma, has an historical importance which renders it worthy of distinct mention. It was a great favorite with the Greek sophists on account of its affording a convenient cover for a fallacy or sophism, through the complexity of its elements. It is a purely hypothetical syllogism, as the reasoning does not turn at all upon the disjunction which appears in the sumption, but exclusively on the dependence of the consequent member upon the antecedent; as sumption, *whether A is B or is C or is D, E is F;* subsumption, but *A is B or is C or is D;* conclusion, *therefore E is F.*

Whether the substance spoken of is solid, or liquid, or gaseous, it must be elastic; but it is either solid, or liquid, or gaseous; therefore it is elastic.

The dilemma may proceed in the ponent or in the tollent mode. Moreover, the disjunction may be in the antecedent or consequent of the hypothetical sumption.

We thus have the four forms of the dilemma:—

1. *Ponent with disjunct antecedent; as whether A is B or C, D is E; but A is B or C; therefore, D is E.*

2. *Tollent with disjunct antecedent; as if A is B or C, D is E; but D is not E; therefore A is neither B nor C.*

3. *Ponent with disjunct consequent;* as *if A is B, C is D or E; but A is B; therefore C is D or E.*

4. *Tollent with disjunct consequent;* as *if A is B, C is D or E; but C is neither D nor E;* therefore *A is not B.*

See Appendix No. VIII.—Exercises in Hypothetical Reasonings.

§ 96. A DISJUNCTIVE SYLLOGISM is a mediate reasoning in which a new judgment is derived from a disjunctive judgment through a judgment removing the disjunction; as *A is B or C; but A is B; therefore, A is not C; the sponge is either mineral or animal; but it is not mineral; therefore it is animal.*

It is governed by the first pair of the fundamental laws of thought:—the law of disjunction and the law of exclusion. Its principle is that under these laws *positing one disjunct member of a judgment necessarily sublates the other; or sublating one posits the other.*

§ 97. The disjunctive syllogism is composed of three judgments. The sumption is a disjunctive proposition and must be affirmative from

the necessity of thought, which under the law of disjunction must either affirm that the terms are identical or are not identical.

The subsumption removes the disjunction presented in the sumption. It may do this either by positing or by sublating either one of the disjunct members.

The conclusion may be either affirmative or negative. If the subsumption posits one member, the conclusion sublates the other. This is the affirmative or ponent mode. If the subsumption sublates one member, the conclusion posits the other. This is the negative or tollent mode.

EXAMPLES:—Common sumption; *The ancients were in genius either superior to the moderns, or inferior, or equal:*

PONENT MODE:—Subsumption; *But the ancients were superior to the moderns;*

Conclusion:—*Therefore they were neither inferior nor equal.*

Or

Subsumption:—*But the ancients were either superior or equal to the moderns:*

Conclusion:—*Therefore they were not inferior.*

TOLLENT MODE:—Subsumption; *But the ancients were not inferior.*

Conclusion:—*Therefore they were superior or equal.*

Or

Subsumption:—*But the ancients were neither inferior nor equal.*

Conclusion:—*Therefore they were superior.*

See Appendix, No. IX.—Exercises in Disjunctive Reasonings.

§ 98. Syllogisms have been distinguished into different kinds in respect to the form in which they are stated. The form of statement of course does not affect the validity of the thought; but for the sake of brevity or for other reasons in discourse the mode of statement is generally different from the proper, full and formal modes in which the syllogism is set forth in logical systems. All of these popular modes of statement if valid can be reduced to the regular logical forms, and it is the chief object in these systems to exhibit the true characters of all valid movements of thought in such forms in order that the looser modes of actual discourse may be easily reduced to them and so be verified by them.

Irregular syllogisms are of three kinds:—1. *Incomplete* as already exemplified in the Enthymeme, in which one premise is omitted (§ 84); 2. *Complex;* and 3. *Regressive.*

§ 99. Complex Reasonings consist of two or more reasonings combined. Immediate reasonings are thus often combined with one another and with mediate reasonings: and mediate reasonings are combined with one another in every possible variety of combination. Thus in the

reasoning: *all birds are winged; therefore some winged animals are birds*, we have the two immediate reasonings of conversion and restriction combined. This complex reasoning is technically called *partitive conversion*, also *conversion by limitation* and *conversion per accidens*.

The reasoning: *all mammals are vertebrates; therefore no invertebrate is a mammal*, is complicated of conversion and transference. It is technically called *conversion by contraposition*.

The reasoning: *All mammals are vertebrates; whales are mammals; therefore some whales are vertebrates*, is complex of a deductive syllogism and restriction.

In the following example, we have a complication of two tollent hypothetical reasonings:—*If men were prudent, they would act morally for their own good; if benevolent, for the good of others; but many men will not act morally, either for their own good, or that of others; some men, therefore, are neither prudent nor benevolent.* In a full and separate statement we should have first the hypothetical syllogism: *if men were prudent they would act morally for their own good; but many men do not act morally for their own good; therefore many men are not prudent.* Then we should have a second with the conclusion, *therefore many men are not benevolent.* Finally, having judgments with the same subject, we obtain by composition (§ 77) the conclusion.

§ 100. Sometimes the reasoning in discourse

contains more than one simple reasoning, each following syllogism depending on the preceding. It is then called a *Chain of Reasoning* or a *Polysyllogism* as distinguished from a single syllogism called for the sake of distinction a *Monosyllogism.* Thus as it is unnecessary to state the proposition which forms the conclusion of one syllogism and the sumption of a second, we may abbreviate the statement by combining into one connected chain of reasoning the two syllogisms. Thus in the two syllogisms, 1. *An animal is a substance; a quadruped is an animal; therefore a quadruped is a substance;* 2. *A quadruped is a substance; a horse is a quadruped; therefore a horse is a substance;* the statement of the proposition which is the conclusion of the first and the sumption of the second syllogism is unnecessary, as it is readily supplied in thought. We have thus the chain or polysyllogism: *An animal is a substance; a quadruped is an animal; a horse is a quadruped; therefore a horse is a substance.*

The Polysyllogism may contain two or more single syllogisms. There will be as many single syllogisms always in every chain of reasoning as there are middle terms if the polysyllogism is categorical, or as there are mediating judgments if it is conditional. Thus in the example given, there are two single syllogisms as there are two middle terms—*animal* and *quadruped.*

§ 101. Sometimes in statement one of the syllogisms is connected with one of the premises.

Thus in the polysyllogism, *an animal is a substance; a horse is an animal for it is a quadruped*, we have the statement abbreviated by connecting one of the syllogisms immediately with the premise *a horse is an animal*, by the conjunction, *for it is a quadruped.* Such a form of statement is called an *Epichirema* to distinguish it from the regular form of a chain of reasoning which is called a *Sorites.*

A SORITES may accordingly be defined to be a polysyllogism in which the single syllogisms are stated in like relation to the whole series.

AN EPICHIREMA is a polysyllogism in which one or more of the single syllogisms is attached to a premise in one of the others.

§ 102. Moreover, the movement of thought may be stated as advancing from consequent to antecedent as well as in the more regular order which is from antecedent to consequent. Thus in the example given, *an animal is a substance; a quadruped is an animal; a horse is a quadruped; therefore a horse is a substance*, we have a movement from antecedent to consequent. This movement is for the sake of distinction called the *Progressive Reasoning.*

The other statement which presents the consequent first is called the *Regressive Reasoning;* thus—*a horse is a substance, for it is a quadruped; and a quadruped is an animal; and an animal is a substance.*

The Progressive reasoning is characterized by

the conjunctions *therefore*, *consequently*, and the like; the Regressive, by the conjunctions *for*, *because*, *since*, and the like.

The sorites, it will be noticed, is a polysyllogism which is purely progressive or regressive; the epichirema introduces a regressive movement into a chain otherwise progressive.

See Appendix, No. X.—Exercises in Irregular Reasonings.

PART II.

THE METHOD OF THOUGHT.

CHAPTER I.

INTRODUCTION.

§ 103. THE logical doctrine of Method presents the conditions of perfect thought in order to its end, which is truth, science, or perfect knowledge.

Thinking in man is a function of a rational nature and consequently implies an end—an end worthy of a rational being. This end can be no other than truth. We think in order to know. Perfect thought is certain knowledge, if the *datum* of thought be true; for thought deals only with what is given it, with what is apprehended. It is one thing, however, to know the laws of thinking and the possible forms of thinking and quite another thing to know how to use these forms for attaining the rational ends of thought. A knowledge of method is requisite—of the way to use the capacities of knowledge for the attainment of certain knowledge or truth. Logical science in this department appears more

characteristically as a normative or regulative science. Practical skill in thinking prescribes in addition exercise in conformity with the true nature of thought in its divers forms and movements and with the conditions of thinking so as to attain the ends of proper thought. Skill comes only in the union of practice with intelligence of powers and methods. Habitual skill in thinking requires still further beyond the limited examples for practice that text-books give, the training of the mind on the ordinary occasions of life in accordance with the principles of right thinking. Logical systems at their best can only afford the needful suggestings and promptings, the guides and helps to this discipline of thought.

There may be recognized, indeed, a diversity of ends in logical training, which will so far modify the principles of logical method. The ancient Greek logicians had their systems of *dialectics*, the logic of the probable, according to Aristotle; their *eristics*, also, or the logic of disputation, the end being victory; and *apodictics*, or logic of perfect or necessary thought, the end being truth for its own sake. It is this last mentioned treatment of the science, as a normative science of thought for the attainment of truth for its own sake as its end, with which we have to do.

§ 104. The conditions of perfect thought are threefold :—

1. *Material Conditions*, or those founded in the

matter of thought constituting the *Objective Law of Thought;*

2. *Formal Conditions*, or those founded in the nature of thought itself, constituting the *Subjective Law of Thought;* and

3. *Verbal Conditions*, or those founded in language as the instrument of thought, constituting the *Verbal Law of Thought.*

§ 105. Inasmuch as in thinking we may aim to produce new thought which shall be true, or verify an already finished product of thought, logical method will have a twofold procedure; one in producing new thought and the other in verifying a product of thought. The one will properly be the reverse of the other. Although the movement of thought is thus diverse, and the distinction should in order to perfect thought be kept in mind, it will not be necessary, except in the single case of induction, to determine and set forth separately the specific modifications of the laws of method to suit each procedure. The two procedures are united in all extended thinking. Even in the simplest form of thought, the *datum* must be verified before the mind can safely proceed to form its product. And generally, as in the solution of a mathematical problem, the verification must, in order to the fullest assurance of correctness, attend each step of the process in advancing to the resultant truth.

§ 106. The matter of thought is, as we have seen, presented to it—is ever a proper *datum*—

and accordingly it is ever assumed by thought. This *datum* to thought may be the product either of the presentative faculty—a perception or intuition—or of the proper faculty of thought. It may be the product of either faculty exclusively, or of both together.

It does not properly belong to logical science to verify these presentations of our perceptive and intuitive faculties; to show that they exactly conform to their objects or to the reality of things. But as the object of thought is not mere empty forms but substantial truth, logic must assume that our perceptions and our intuitions are trustworthy and that they give us in the main true cognitions of the world around. It must assume at least that there is a real world corresponding in certain respects to our own intelligence, so that when we observe and reflect according to the laws of our minds, our observations and reflections are conformed to the realities which they appear to regard. Logic, thus, as it revolts from the idea of being a mere phantom and unreal show, assumes that there is a universe of being around us into which our thought may enter and find everywhere what corresponds to its own nature; that this universe is but the treasure-house of the same kind of thought that our own thinking nature produces, being governed by the same laws and having the same characteristics. It is only upon this as-

sumption that any true knowledge of external things can ever be attained by us.

It follows from this that inasmuch as thought is in its very nature *discursive*, discriminating in every object of knowledge, in every *datum* to thought, subject and attribute which it yet recognizes as one and the same and so ever identifies, everything that can be known or thought by us must be accepted as admitting in its nature this discrimination and this identification. Every object that we can know may in other words rightly be viewed as subject having an attribute, which subject and attribute are inseparable and the same.

Further, the diversification which belongs to the selective nature of thought, implies that in every object of thought there is a diversity of attributes; any one or any number of which may in a given thought be identified with the subject. What precisely these attributes are in any case, are not determined by thought, but are given to it; while out of the diversity thus given the faculty of thought may select such as may subserve its purpose at the time.

Every object then that can be thought must be a unit and also a diversity—a single subject having a diversity of attributes—a whole having parts.

§ 107. Attributes presented as belonging to the same subject are said to be in logical *congruence*—are *congruent* parts; while attributes that

are not capable of being thought to belong to the same subject are in logical *opposition*—are *opposites*. Of this logical opposition in the matter of thought, besides the pure opposition in thought itself, we have distinguished the two degrees of contradictory and contrary opposition (§ 37).

It follows that in order to perfect thought or true knowledge its object must be apprehended as one and as having congruent attributes.

This logical principle of congruence has a wide significance and application. Whenever in the movements of thought we bring in that which is incongruous, and the movements proceed in this incongruous matter or form, as they may with perfect legitimacy in themselves to a greater or less extent, the results become illusive, self-contradictory, or absurd. Some of the familiar logical puzzles illustrate this truth. That of Achilles and the tortoise thus involves the incongruous notions of time as continuous and also at the same time as discontinuous. The mathematical puzzle that the quotient arising from dividing zero by zero is seven [$\frac{0}{0} = 7$], and proved by the multiplication of the quotient into the divisor [$7 \times 0 = 0$], involves the incongruity of the notion of a positive quantity combined with the notion of the utter negation of quantity, or the incongruity of the non-quantitative treated as the quantitative.

Still further, in order to true knowledge, the object must be apprehended just as it is, both as one whole and also as to its diverse parts;—it must be *adequately* apprehended, so that in the thought it shall be exactly conformed as a whole in all its boundary and extent to the object; and *accurately* apprehended, so that each part shall be conformed in itself and its relations to the corresponding part in the object.

Such are the three material conditions of perfect thought; 1. *Unity and objective congruence* in the object itself as apprehended and the parts of it regarded in the thought; 2. *Adequateness*, or conformity of the thought to the object as such a whole; and 3. *Accurateness*, or conformity of the thought to the parts severally in their own nature and in their relations as parts of one whole.

§ 108. The formal conditions of perfect thought are threefold: 1. That it truly identify or difference; 2. That it be clear; and 3. That it be distinct.

The first condition named respects the essential function in all thought;—it must identify or difference. The perfection depending on this condition is called from the positive side—*Harmony* or logical *Congruence*. It is of course but the thought-side of the first material condition mentioned (§ 107).

The second condition—*Clearness*—respects the nature of thought as a product of the intelligence

when its object is viewed as a single whole by itself separated from all other objects.

The third condition—*Distinctness*—respects the nature of thought when its object is viewed as a whole consisting of parts.

Thus I have a *clear* thought of the earth when I view it as one whole completely separated from all other objects. I have a *distinct* thought of the earth when I view it in its several parts each separated from the others and in its just relations to them.

This distinction becomes of important use in the treatment of concepts.

§ 109. When the formal conditions of knowledge are observed, we have so far necessary truth (§ 43). The distinction between necessary and contingent truth lies primarily and originally in this—that all legitimate movements in thought are characterized as necessary, being such that the opposite must be regarded as absurd or as in contradiction to the fundamental principles of knowledge; while contingent truth is that which is reached in some mode not subject to the necessities of thought.

Pure, legitimate thought gives ever necessary truth; contingency comes from the matter of thought or from irregular thought. The truth of perceptions and of intuitions, thought accepts; but the fruit of observation or of experience does not bear the character of necessity, like the pure movements of thought. If a field is sixty rods

long and forty rods wide, the content is necessarily twenty-four hundred square rods. The calculation in thought does not verify the observation and the measurement; it only accepts them as possible. The statement of the content is a contingent truth—probable or possible—simply because the matter of the thought is contingent. Mathematical truth is necessary because it is the result of pure thought moving upon matter that cannot be questioned. Its *data* are suppositions or definitions; and the movements of thought upon these *data*, accepted as true beyond contradiction, give results that are equally beyond contradiction—are necessarily true. Surveying and astronomy being grounded on observation, are contingent sciences, because the original *data* to thought being observations are not verified in thought. As the matter of thought becomes verified in thought, the contingency approximates to necessity.

It should be remarked, however, that the fallible mind of man is liable to err even in pure thought; its results, in so far as subject to this possibility, are characterized as contingent. Still farther, the use of language as the necessary instrument of extended thought and its final embodiment, is subject to error; and is consequently an element of contingency in knowledge.

§ 110. So far as the movement of thought itself is concerned, the character of the *datum* to which the thinking is applied can make no differ-

ence in the result. Whether the supposed field be a reality or not, sixty rods of length and forty rods of width give ever a superficial content of twenty-four hundred square rods. Yet, as we think in order that we may know the truth, it is not impertinent in determining the method of thought in order to perfect knowledge, to indicate the necessity of a careful attention to the character of the *datum* or the matter of which we think. If our aim in thinking be real knowledge, then the matter with which we start must be real; —it must have been legitimately given to our thought;—it must be a true apprehension. This is the first condition then in all thought that aims at real truth, that its *datum* be a true apprehension verified by the signs and characteristics by which presentative knowledge is to be accredited. Is this *datum* an actual perception, not a phantasm, or is it an actual intuition, not a fancy; and is it a pure perception or intuition unmixed with other impressions or conceits; or is it a product of pure thought that has legitimately sprung from such a perception or intuition? If the *datum* be real, the result of any valid thought movement upon it will be real also; if the *datum* be imaginary or of doubtful reality, the result of the thought-movement can have no higher character.

§ 111. The verbal conditions of perfect thought are *Correctness* and *Perspicuousness*. The words used as the embodiments or as the instruments

of thought, must correctly express the thought and also perspicuously signify what is meant, or the thought itself must falter or wholly fail. It will be seen that a large proportion of the fallacies in thought are to be traced to a faulty or deceptive use of words.

Sir William Hamilton has well observed that while language must ever follow thought, inasmuch as it is indeed the creation of thought, yet thought itself can proceed but a very little way without the support of language. He well likens the reciprocal relation between thought and language to that between a tunnel and its arch. The tunneling must begin before the arch can be supplied; but it cannot advance far without the support of the arch.

He justly advances, moreover, the following postulate, the only postulate of logic, he says, which requires an articulate announcement; that logic "be allowed to state explicitly in language all that is implicitly contained in the thought." If, for illustration, the proposition *man is mortal* implies in the thought that *mortal* is but a part of what may be predicated of *man*, logical interpretation may be allowed to state this and require the explicit statement: *man is some mortal.*

The converse of this postulate may be as justly claimed: that logic be allowed to interpret the verbal expression of thought according to the exact limits of the thought to be expressed. Summarily, logic postulates that the verbal body

of thought exactly correspond with the thought to be embodied—neither express more nor less than the thought intended. This must be the allowed principle everywhere in the use and the interpretation of language,—that the thought govern absolutely.

§ 112. Language embodies in its words or clauses the three different elements of thought,—the subject, the predicate, and the copula,—and distinguishes them by different forms peculiar to each.

Words denoting subjects of propositions are called *Concretes*, and are of four different classes:—

1. *Proper names*, denoting single subjects, as *John*, *Venus*, *Malta*.

2. *Class names*, denoting generic concepts, as *man*, *star*, *island*.

3. *Collective names*, denoting wholes composed of individuals or numerical parts as groups and not as classes; as *army* composed of soldiers, *forest*, *multitude*.

4. *Mass names*, denoting wholes composed of spacial parts, as *air*, *water*, *body*.

§ 113. Words denoting attributes called *attributives*, also *connotatives*, are of the four classes corresponding to the four classes of attributes mentioned: *Qualities*, *Actions*, *Conditions*, *Relations*.

As attributes themselves may be thought as subjects of which other attributes are predicated,

language gives to them the form of names and thus presents a class called *Abstracts*, which are distinguished from concretes, as original attribute words are distinguished from original subject words.

CHAPTER II.

METHOD OF THOUGHT—THE JUDGMENT.

§ 114. In order to truth in its largest sense, an inquiry preliminary to the application of the proper conditions of perfect thought to any one of its special forms will naturally arise in respect to the character of the *datum* itself. Is that real, or imaginary? Is it pure and unmixed? Is it embodied in words that accurately and perspicuously express it?

Farther, in order to objective truth, that is, a perfect knowledge of things about us as real, it must be assumed that these things about us, so far as objects of our thought, must admit of being known under the relation of subject and attribute. A judgment giving objective truth must assume that its *datum* may be correctly viewed as a whole or a subject, having parts or attributes; and moreover that these attributes while diverse are congruously united in this one whole or subject.

§ 115. Of the material conditions proper to thought, the first respects the unity and congruence of the *datum* or matter. The necessities of valid thought in a judgment require that if a plurality of objects are contained in the subject or a diversity of attributes in the predicate, they

should be so far congruent as to be capable of being thought as one whole. The incongruousness which is forbidden often lies concealed in particulars which escape notice. The word *centaur* denotes a union of man and horse, which is possible perhaps in some particular attributes, but utterly inconceivable in others. There are characteristics of the two, such for instance as *a man's mouth* and the *food necessities of a horse*, which are entirely incongruous, so that no perception could take in such a union. Although many things might be correctly thought of such a *datum* while the view was confined to congruous elements, as soon as the movement of thought has turned upon those that are incongruous —those that cannot be joined in one object of perception—the whole procedure becomes fallacious, and the result is at least doubtful, perhaps unreal and false. The name *Hercules* represents a mythic character to whom in different myths are ascribed feats and experiences which are entirely irreconcilable in one person. To accept such a character as a true *datum*, and then to found upon it movements of thought however legitimate in themselves, can lead at best only to uncertainty, perhaps even to results empty of truth or monstrous and absurd.

§ 116. The second material condition of perfect thought—*adequateness*—requires that the subject and the predicate in the *datum* be each apprehended in thought as neither more nor less than

as given. If, for illustration, the notion of *science* as presented in the technical definition, *Logic is the science of thought*, be the *datum*, and a judgment be made by predicating of it such an attribute as *sentient*, the notion would be taken in a sense broader than its proper technical use in the definition admits; for *science* in this use denotes reflective and systematic knowledge, which character does not belong to merely sentient knowledge. This is a most vital principle of logical method, specially in the form that the judgment must never transcend what is given to it. It must not put anything new into the *datum*.

§ 117. The third material condition of perfect thought—*accurateness*—requires that the *datum* be apprehended as to its parts just as it is. Thus in the proposition, *thought is intuitive*, the judgment is inaccurate; for *intuitive* is not one of the proper parts of thought—not one of its true attributes.

The virtue of adequateness in a judgment properly and originally respects the subject; for in a primitive judgment the subject is taken as a whole and the predicate is taken as an attribute, that is, as a part of the subject. For the same reason the virtue of accuracy properly and originally respects the predicate. But derivatively nothing forbids that the judgment should be viewed throughout as in extensive quantity, and then the subject is a part of the predicate. Thus the proposition *the horse is an animal*, may in accord-

ance with every principle of thought, be construed in the sense that *the horse is a part of the animal kingdom.* It is as legitimate as the proposition, *the horse is animal.*

A similar remark may be made of such judgments as are made up of a subject and predicate separately given. Thus, while such propositions as *the sun is bright* are to be regarded as primitive and as the normal types of judgments, yet such propositions as *the sun is center of the solar system* are equally legitimate, although here the *datum* is not presented as a single object in which thought afterwards distinguishes subject and attribute, but as twofold, coming into the thought from separate directions. The two are united—identified in thought as subject and attribute—the latter being recognized as a part of the whole sphere of attributes pertaining to the subject.

The Objective Law of a judgment accordingly enjoins (1) congruence in the parts of the object thought; (2) adequateness, or exact correspondence in the thought to the object as a whole; and (3) accurateness, or exact correspondence in the thought to the several parts of the object.

§ 118. The Formal conditions of perfect thought, that it truly identify or difference, that it be clear, and that it be distinct, as also the verbal conditions of correctness and perspicuousness, need no further illustration in their application to the general forms of a judgment.

The conditions of perfect thought as applied

specially to several of the divers kinds of judgments demand farther consideration.

§ 119. The Disjunctive judgment thus presents peculiar difficulties. Its essential nature consists in its *disjoining;* it must actually disjoin or it loses its character. But this disjunction, as has been shown, may be of divers kinds or degrees. We have thus pure contradictory opposition, when the judgment disjoins the two forms possible to thought—the identifying or the differencing—the affirmative or the negative; as in the formula *A either is B or it is not B.* If these two, A and B, come into the thought, and be viewed in their relations to each other as subject and attribute, it is of the very necessity of thought that it recognize them as the same or as different. This is the typical form of a disjunctive judgment (§ 36).

But we have the form *A is either B or non-B, the stone is organic or inorganic,* in which the disjunction or opposition is not in the thought itself but in the matter—the predicate. This is called contradictory opposition, but it does not bear the character of perfect thought and does not give necessary truth. If we substitute another predicate in the example given we shall have an untrue judgment; thus, *the stone is either vertebrate or invertebrate.* This form of disjunctive judgment then must be reducible to the form of a pure contradictory judgment before it can be accepted as necessarily valid. This may be done

if the disjunct members, *organic* and *inorganic*, can be recognized as true disjunct alternatives; that is, if the judgment can be reduced to the form of a pure contradictory—*the stone is organic or is not organic*—by showing that there is no third member admissible in thought.

In the form *A is either B or C, the line is either straight or crooked*, it becomes necessary in order to verify the judgment as of necessary truth, to recognize *C* as the true alternative of *B*, *crooked* as the true alternative of *straight:* then it is to be verified as before.

In the form *A is B or C or D, angles are right or acute or obtuse*, still another step is requisite in the verification. It is necessary first to recognize *A as B or non-B, angles as right or not right*, and afterwards to recognize *non-B* as *C or D, not right* as *acute or obtuse*.

In short, a disjunctive judgment in the lower forms of opposition is to be verified by a reduction to the form of a pure contradictory opposition—a proper disjunction in thought. The proper test of this is that the two disjunct members are necessary alternatives, so that if either cannot be accepted the other must be (§ 36).

The disjunctive particles, *either—or*, *neither—nor*, *whether—or*, are ambiguously used to express a disjunction in the thought or in the form of expression. It is necessary, therefore, carefully to see that the disjunction be unambiguously in the thought; otherwise the proposition is not a logi-

cal disjunctive. Thus, there is ambiguity in the propositions, "*His book will be condemned by verbal critics or grammarians:*" "*A class is a genus or species.*"

Moreover, the disjunctive *or* sometimes opposes congruent attributes; thus, "*A good horse is fleet or strong.*" It is used still farther as a true copulative, where the attention is to be directed to separate views; as "*The ruins of Greece or Rome are but the monuments of her former greatness.*

§ 120. In the class of judgments distinguished in respect of their modality, the liability to error is twofold. The first lies in an unlawful substitution of a lower for a higher degree, as of a simple assertory or a problematic for an apodictic judgment. The second lies in an unlawful transference of the modality from the copula to the matter. The judgment should be recognized as expressing no other modality than is intended.

§ 121. Most judgments in discourse are partial judgments in which the subject is but part of the predicate, as "*the whale is a mammal,*" or, as is still more common, the predicate is but one—is but a part—of the attributes belonging to the subject, as *The whale is warm-blooded.* It becomes necessary, therefore, to recognize fully in partial judgments that only a part and also what part is thus expressed in the judgment. A partial attribution is often taken for a full definition—that is, for an identical judgment. Thus the

proposition "*Life is the coördination of actions*," is true as a partial judgment; but if interpreted as an identical judgment or an exact definition, it is not true; for then crystallization would be life, as it is a coördination of actions.

§ 122. The peculiarity of the Hypothetical judgment is that the terms are judgments. It is necessary therefore in these judgments to recognize the antecedent as a judgment which involves the truth of the consequent. Farther, inasmuch as one truth may involve another, either by its being a whole of which the consequent is a part, or by its being a part of which the consequent is a complementary part, it is necessary to recognize which of these relations subsists between the terms. Thus the proposition, *If men were prudent they would act morally for their own good*, is verified by recognizing that the consequent *acting morally for their own good* is contained as a part in the more comprehensive judgment expressed in the antecedent, *if men were prudent*. It is verified in other words by analyzing the antecedent and finding the consequent among its parts, as that *moral* is a part or attribute of *prudence*.

But in the proposition, *If all the planets that have been observed have elliptic orbits, then those that have not been observed must have elliptic orbits*, the relation of antecedent to consequent is that, not of a whole to a part, but of part to complementary part. The mode of recognizing the

truth of the hypothetical judgment in this case is far different from that in the former.

§ 123. In order to perfect thought, accordingly, in a judgment, we have, to express them summarily, the following conditions to observe :—

1. As to its quality, to recognize it whether as identifying or as differencing ; or if in disjunctive quality, as logically disjoining and whether in pure contradictory, contradictory, or contrary opposition ;

2. As to its modality, to recognize it as assertory, problematic, or necessary ;

3. As to degree of identification, to recognize it as total or partial ;

4. As to the logical form of its matter, to recognize it as categorical or hypothetical; and if hypothetical, whether it expresses the relation of whole to part or of part to complementary part ;

5. As to verbal expression to recognize it as corresponding to the thought—that it be correct —and also that it be perspicuous.

See Appendix, No. XI., Exercises in the method of judgments.

CHAPTER III.

METHOD OF THOUGHT—THE CONCEPT.

§ 124. As the concept is an organic constituent of the judgment its *datum* can be fully verified only through the organic whole of which it is a member. In order to be assured thus that a class-concept is true to what it represents, it is necessary to inquire first: Are those judgments on which the concept is founded true—have these several subjects which are united in the concept really the common attribute predicated of them? Or in case of the comprehensive concept, do the several attributes predicated of the common subject actually belong to it? It will be remarked that the composite concept cannot from its very nature have its exact counterpart in the external world either in any group of objects standing apart from all other groups, each possessing only the attributes upon which, as common to them all, we have gathered them into a class; nor in any aggregate of attributes combined in some one object and no other. Each individual, for instance, in the class of objects which we have synthesized into the extensive concept *planet*, has manifold other attributes than those which have furnished the base of the class;

nor is there anywhere to be found an aggregate of attributes existing by themselves exactly corresponding to those united in the comprehensive concept *animal*. Our vocabulary of concepts does not in fact contain exact images of external realities any more than does a case of type contain the exact images of the several words in a discourse. It would be as absurd to seek in concepts the exact forms of things without, as in the boxes of type to find the exact forms of words in a vocabulary. They spell all words, all things, but are not themselves the exact images of any words, any things.

In order, therefore, to verify our concepts as true representatives of external realities, we have only to seek whether there are realities brought to our thought through our perceptions as trustworthy channels which, when they are reflected upon, reveal themselves as possessing in common among other diverse attributes, the simple attribute or the aggregate of attributes on which the class is formed; or in the case of a comprehensive concept, whether there is the simple object given to our thought through our perception which possesses those several attributes that are combined in the concept. Unless there are objects external to us which have some one attribute in common, there can be no class-concept that truly represents the outer world; and unless divers attributes which we may distinguish in our thought are in some one object united together so as to

form a congruent unity to our thought, then our attribute concepts are mere phantoms. If, regarding the real world of objects and of events around us as a veritable creation uttering so many words of the Divine Creator and Ruler, we find our concepts spelling out rightly these words so far as we have observed them, then we are justified in accepting them to be valid and trustworthy, just as we would accept a font of type that spells out every word to which we apply it to be correctly made and correctly distributed in the case.

§ 125. We frame our concepts as we plan our fonts of type, according to the use we intend. As we have Greek fonts for Greek words, Roman for Roman, German for German, and moreover, italic and accented characters and the like for our divers objects in printing, so we frame concepts for different uses in knowledge. The astronomer groups the heavenly bodies into a peculiar class of concepts for his purposes; the poet groups the same objects into another class for his uses; the adoring worshiper into a third for his. The simple explanation is that each regards different attributes of the same objects, and so builds his classes on different foundations. In verifying as also in using these concepts we need to bear this in mind, else we fall into the same error and folly with him who should indiscriminately use Greek type to spell English words. The universe has not been distributed

into just such classes as we may for our uses be pleased to construct; and the attributes are not all collected into the same number and kinds of unions as those to which our uses lead. The concepts are true to the world about us, as we have said, if they can be used to spell out correctly every ordinance of heaven to which they refer.

§ 126. Passing now to the more strictly logical principles of method in forming and verifying the concept, the material conditions of perfect thought, constituting its Objective Law, prescribe in the first place that the terms which are united to form the concept be recognized as homologous terms, that is, as either subjects or predicates of judgments having the same base. If it be a subject or class-concept—an extensive concept—then all the terms that are united must be subjects and they must all have the same predicate. If it be a predicate or comprehensive concept, then all the terms that are united to compose it must be predicates and must all belong to the same subject. This is the law of *unity and congruence.*

In the second place, this Objective Law of the concept prescribes that the base of the concept, which is the common subject or the common predicate in the primitive judgments, be exactly recognized in its extension or its comprehension. This is the law of *adequateness.*

In the third place, this Objective Law of the

concept prescribes that the terms to be united into the concept be exactly recognized each in its extension or comprehension. This is the law of *accurateness*.

§ 127. The Formal conditions of perfect thought constituting the Subjective Law of the concept prescribe in the first place that the aggregation of the terms make up one congruous whole.

This law is the exact counterpart of the first objective law just stated; it is simply subjective, while that is objective; this looks at the thinking subject, that at the object thought. As the object, in order to be thought, must be one whole whose parts are congruent, so the thought of the object must actually recognize it as one and congruous.

This law has its application in the amplifying of concepts, the two processes of which being *Generalization*, or the amplification of an extensive concept, and *Determination*, or the amplification of a comprehensive concept.

§ 128. Generalization is effected by combining subjects having the same attribute (§ 56). Thus beginning with an individual, as *Socrates*, given as subject in the judgment *Socrates is a citizen of Athens*, by combining *Plato*, *Xenophon*, *Aristotle*, subjects in like judgments, we form the class *Athenian*. By combining *Spartan*, *Theban*, with *Athenian* in such judgments as an *Athenian dwells in Greece*, we obtain the higher class *Gre-*

cian. In similar ways we may rise to still higher gradations, as *European*, *terrestrial.* Language will of course often fail to furnish already coined words for naming classes which are possible, since these are without limit.

The attribute in respect to which as the base the generalization proceeds is called the *principle of generalization.*

The logical principle of congruence in generalization simply requires that only those subject-concepts be combined which have the same attribute. But in generalizing for some use in positive knowledge, two leading things to be done as determined by this use are: first, to select the attribute on which we shall form our classes, and secondly, to fix upon the number of gradations in our proposed classes. These determinations lie outside of the proper province of pure logic. In the various sciences we find systems of classification varying with the proposed uses of the particular science and the character of the subject-matter of the science. Pure logic recognizes only the three gradations of *individual*, *species* a group of individuals, and *genus* a group of species. Accordingly it may happen that a class may be a genus considered in relation to lower classes while it is a species in relation to higher. *Grecian*, thus, is a genus in relation to *Athenian*, but a species in relation to *European.* But in science of objects as real, the designations be-

come more fixed, being determined in reference to actual attributes.

Natural History thus has furnished very precise and fixed systems of classification, as it has sought to ground itself almost exclusively on the purely objective attributes, with little reference to special use or occasion. The departments of this science which treat of organic being employ the term *species* in the more definite meaning to denote a class of living things that are capable of permanent propagation or that have the same ancestry. It is founded on the principle of *kind* in the natural world, by which like produces like —the same produces the same.

In Zoölogy Agassiz proposes the following gradation of classes:—

Types or Branches, founded on a plan of structure;

Classes, on the manner of its execution;

Orders, on the greater or less complication in execution;

Families, on form;

Genera, on details of structure by which members of the same family differ from one another;

Species, on differences in the preparation of the parts and in the absolute size of the whole animal, in the color and ornamentation of the surface of the body, and in the relations of the individuals to one another and to the world around.

In Botany the leading gradations adopted by naturalists are *Series*, *Class*, *Order or Family*,

Genus, *Species*. Under these respectively are divers subdivisions as *sub-class; sub-order*, *tribe*, *sub-tribe; sub-genus or section;* and under species, *variety*, *sub-variety*, etc.

In all the sciences the systems of classification are subject to change as in the progress of knowledge new observations are made or new relations of science arise. Still the pure logical principle applies everywhere, as do the principles of pure mathematics, in all diversities of subjects.

The principle of congruence as applied to generalization through divers degrees requires that objects of different grades be not indiscriminately mingled together.

The Rules of Generalization accordingly are:—

1. Determine the attribute in the given subject which is to be the principle of generalization;

2. Add to the subject given to be generalized other subjects which have the attribute selected as the principle of generalization;

3. Arrange coördinate subjects together, and place subordinate subjects under their respective immediate superordinates.

§ 129. DETERMINATION or the amplification of a comprehensive concept is effected by combining attributes belonging to the same subject (§ 58). Thus to the attribute of Socrates as *observing* we may add the attribute of *reflecting*, and we obtain the ampler concept of *philosophizing;* and by adding the attribute of *teaching*, we have the still more comprehensive concept of *teacher of philos-*

ophy; and by adding the attribute of *moralizing*, we have the concept of *philosophical moralist*. The process is essentially but adding to a given attribute any other attributes belonging to the same subject.

Language does not always furnish single words for denoting amplified attribute-concepts so as to distinguish them from the corresponding subject-concepts. Thus if to the attribute *round* as an attribute of the *sun*, we add another attribute—*bright*—we have the amplified concept *bright-round*, not expressed by any authorized word in our vocabulary. If we were to add attributes of relation as *in the heavens, center of motion to planets*, etc., the poverty of language to express our amplifications would be more conspicuous.

This process is the exact analogue of generalization. By amplifying the attribute which marks a Series in Botanical Science, we reach the lower grade in generalization, called a Class; thus adding to the attribute *flowering*, that of *endogenous*, we reach one of the two classes into which flowering plants are distributed; adding characteristics of structure, we reach the lower grade of Orders.

The rules of Determination accordingly are:—

1. Determine the principle of determination, which is ever some subject to which the attribute given to be amplified belongs;

2. Add to the attribute given to be amplified

other attributes found in the subject which is selected as the principle of determination;

3. Arrange coördinate attributes together and place involved attributes respectively under those immediately comprehending them.

§ 130. The Subjective Law of method in the concept prescribes in the second place that the thought be *clear*.

Clearness respects a concept viewed as a whole. It is the subjective counterpart of the objective quality of *adequateness*. The process for attaining clearness is termed by logicians *Definition*.

Logical Definition is accordingly that process in which a concept is discriminated from all other concepts.

It is analogous to the geographical bounding of a territory or tract of land. It limits its object from all other like objects.

It is necessarily positive; for a mere negation does not discriminate from all other objects. To say that *Pisidia is not Parthia*, or that *a whale is not a fish*, or that *a stone is not organic*, does not bound or define. Yet it should be remarked that a perfect definition, giving perfect clearness by discriminating from every other object, may, although necessarily appearing in an affirmative judgment, be effected by a negative attribute; thus the definitions that *a brute is an irrational* (*non-rational*) *animal* and that *an oblique angle is one that is non-right*, are logically good, for they clearly discriminate from all other objects.

The old rule of Definition was :—

Name the next higher genus and add the specific difference; thus *man* is defined to be *animal* [next higher genus] and *rational* [specific difference] = *rational animal.* This rule is convenient and generally sufficient to secure the clearness which is the object of definition. But it is applicable only to class-concepts or species. In truth the old logic limited its view almost exclusively to extensive quantity. Its teachings are consequently so far partial and one-sided.

It is not necessary to clearness that all the attributes be named ; hence the rule that the definition must be a strictly convertible proposition is not sound. The converse of *man is rational animal*, viz : *rational animal is man*, is not necessarily true ; for the planets may have inhabitants that are both *animal* and *rational;* but unless they had other attributes, as *two-footed*, *upright*, *without wings*, etc., etc., they could not be classed as *men.* In truth, definition may be effected through the attributes of relation as well as through the essential attributes ; and man can never know that he has ascertained even all the essential attributes of any external object, much less the relative attributes.

The definition of a subject-concept is more properly effected through the attribute or attributes which discriminate it from other subject-concepts ; as *planet* is defined to be a star not

fixed in its position in the heavens; or a heavenly body that revolves around the sun in a slightly eccentric orbit.

The definition of a predicate-concept or attribute is more properly effected through the subject or subjects to which it peculiarly belongs; thus *blue* is defined to be the attribute of color that characterizes the unclouded sky; *rational* to be the attribute which discriminates man from other animals.

Definition, however, is effected with equal facility and logical soundness through complementaries. Thus *radiate* is defined as complementary of *articulate* and *mollusk;* or, according to the old rule, by genus and specific difference as *invertebrate having spheroidal structure.*

A logical definition should be discriminated from a verbal, nominal, or etymological definition, which defines through synonyms or paronyms. Thus, the definitions of *grade* as a degree, and of *gradual* as proceeding by degrees are not logical.

The rules of Definition are :—

1. Recognize the term to be defined either as a subject-word or as an attribute-word.

2. Define subjects by their attributes, and attributes by their subjects; or, define by complementary-concept.

3. Select such attribute for defining subjects as will discriminate them from all other subjects;

and such subjects for defining attributes as will discriminate them from all other attributes.

§ 131. The Subjective Law of method in the Concept prescribes in the third place that the thought be *distinct*.

Distinctness in a concept respects its parts and implies that each of the parts be properly recognized in itself and in its relations to the whole and to each of the other parts. It is the subjective counterpart of *accurateness*. The process for attaining distinctness is *analysis*.

The analysis of a class-concept is called *Division* (§ 59); the analysis of an attribute-concept is called *Partition* (§ 60).

§ 132. DIVISION is effected through the attributes, which form the base of the concept to be divided, simply by addition. Thus, the class-concept denoted by *man* is divided by adding to the attribute *human*, which is its base, the attribute *civilized*, and we thus have the two varieties of the class *man*, *civilized man*, and *uncivilized man* (= *savage*). The necessity of dividing through the attribute lies in the fact that this being the base of the concept is that alone which secures unity. The principle is that a concept and its base, that is, that a subject-concept and its corresponding attribute-concept, vary inversely as each other (§ 61). Thus in generalization, in passing to a higher and larger class, we take away from the attribute; while in

descending from one class to a lower, we add to the attribute.

Inasmuch as to a great extent in language the same term is employed to denote both the subject-concept and its corresponding attribute-concept, we obtain the rule, convenient but scientifically inexact in statement:—*To divide a concept add the attribute which will give one of the parts desired.* To divide *animal* thus in respect to an attribute which will give *man* as one of the parts, we add the attribute distinctive of *man* (*rational*,) and we have the division *rational animal* (*man*) and *irrational animal* (*brute*).

A division into two parts, the one of which must always in perfect division be complementary of the other, is called a dichotomous division—a *dichotomy*. This is the strictest logical division being founded on a pure contradictory opposition; and to it other divisions should be reduced in order to be perfectly verified.

If the division be into three parts, it is called a *trichotomy;* thus *angles are right, acute, or obtuse.* This is verified by two dichotomous divisions; thus *angles are right or non-right; non-right angles are less than right—acute,—or greater than right—obtuse.* Omitting in the final enunciation as unnecessary the second part given by the first division—*non-right*—as it appears in the two parts of the second division, we have the three parts—*right, acute, obtuse.*

Division into more than three parts is called *polytomy*. It is verified by being reduced to corresponding dichotomies. Thus the common division of *vertebrate animals* is polytomous—*mammals*, *birds*, *reptiles*, *fishes*. To verify this by dichotomous division, the procedure would be—1. *Vertebrates are warm-blooded or cold-blooded;* 2. Warm-blooded animals are *viviparous* (= *mammals*) *or oviparous* (= *birds*); 3. *Cold-blooded animals are terrestrial* (= *reptiles*) *or aquatic* (= *fishes*). In order to a perfect verification it is necessary to ascertain by observation that the two members of each dichotomous division are all that exist in nature—that all *vertebrates* that are not *warm-blooded* are *cold-blooded;* that all warm-blooded animals that are not *viviparous* are *oviparous;* that all *cold-blooded* animals that are not *terrestrial* are *aquatic*. The two members in each dichotomy must be exactly complementary of each other, that is, have contradictory attributes.

§ 133. The division will vary with the attribute through which it is made. Thus the division of *angles* into *right*, *acute*, and *obtuse*, respects the attribute of relative magnitude; but a division might be made in respect to any other attribute belonging to angles; thus it may respect the surface on which it is formed and then the division would give angles on plane surfaces (*plane angles*) and *angles not on plane surfaces*, that is on curved surfaces—and a subdivision of these would give *spherical* and *non-spherical angles*. The kind of

attribute thus selected through which the division is to be effected is called the *principle of division.*

As it is the purpose or use for which the division is made that must determine on which of the attributes the division is to be made, the first thing in division is to select the particular attribute which shall be the principle of division.

§ 134. The specific rules of logical division accordingly are:—

1. Select the attribute which shall be the principle of division. This for each division must be single and exclusive. Otherwise no distinctness, but rather confusion would result; as if angles were to be divided into *acute, spherical, obtuse,* etc. This rule does not prohibit the use of a different principle in effecting subdivisions from that used in the higher divisions. Thus *acute angles* may be subdivided in respect to surface into *plane* and *curved.* In truth every successive subdivision will have its own principle.

2. Divide dichotomously. This is done by adding to the attribute taken as the principle of division any congruent attribute. In the final enunciation and use of the parts attained, there is of course no necessity for retaining those parts which are at a subsequent step subdivided into other parts that are named. Thus, as before remarked, there is no necessity of enumerating in the result the higher species of angles *non-right,* inasmuch as its parts are all given in the lower

parts, *acute* and *obtuse*. But to divide *American* into *North American*, *Canadian*, and *South American*, cannot be verified by reduction to dichotomous division, for in such division, *Canadian* would not be opposed to the other parts, or to either of them.

3. Enumerate all the parts. Thus the division of all angles into *plane* and *spherical* is incomplete, for it omits those that are on other surfaces than plane or spherical.

4. Enumerate in order. Subordinate parts must be placed below the next higher respectively, and coördinate parts be placed together. Thus to enumerate the species of angles as *acute*, *right*, and *obtuse*, would violate logical order, for *acute* is of a lower order than *right*, and is coördinate with *obtuse*.

This process is so important and so frequently required in all kinds of thinking, we will exemplify the rules at greater length. We will take the class-concept *man*. We will first divide in reference to one of its essential attributes—the quality *rational*. The first division is effected by adding to *rational* any congruent attribute. We add *cultivated;* and thus have the dichotomous division: 1, *cultivated man;* 2, *uncultivated man* (*savage*). We take next the concept *cultivated man*, and adding some property of action as creating or producing, as *thought-producing*, we have (1) *cultivated man thought-producing* (*thinkers, philosophers, men of science*); and (2) *cultivated*

man producing otherwise than by thought. This last class may be subdivided by adding an attribute distinguishing kind of object produced into (1) *producers of useful objects* (*artisans*); and (2) *producers of other objects*, which can be here only producers of objects of beauty (= *artists*). By adding some attribute of condition as of time to this last class *artists*, we have (1) *ancient* and (2) *modern artists;* or of place (1) *those of the new world* and (2) *those of the old;* which last in like way are further subdivided by more specific attributes of place into (1) *Asiatic* and (2) *non-Asiatic* (= *European*). Still further subdivisions may be effected by adding any of the numerous attributes of relation. Or any one of either class of attributes may be taken as the principle of any step in the process higher or lower, as the purpose or object in the division may require.

While this is the full regular logical method of division, the process, as in arithmetical calculations, may generally be more or less abbreviated; and the mind, when once thoroughly trained, may jump, as it were by instinct, to results, while feeling assured of being scientifically correct.

§ 135. Partition, or the analysis of an attribute or composite-concept, is effected through the subject to which it belongs. It is the exact analogue of division. The subject of the attribute is its base, and accordingly the principle of its unity or identity (§ 60).

As the parsimony of language makes the same

word express either quantity, and as thought has preferably moved in extensive quantity, it is not practicable to find words to exemplify this process in the details of which a division admits, and the procedure itself seems, therefore, at first awkward and unnatural. But the analysis of a composite attribute is a common necessity in thought and discourse; and therefore a facility in effecting it and in verifying is indispensable to the highest promotion and acquisition of knowledge. At all events it should be understood that partition, as the analysis of a composite attribute, is a true and valid logical process, and that any given partition can, if sound, be logically verified.

The logical validity of the process is sufficiently exemplified by retracing any process of division. Thus the attribute *artistic* or *producing objects of beauty*, is analyzed by adding to the class *artists* the class *artisans*, giving the larger class of producers. We find accordingly the dichotomous partition of *artistic* to be *productive* and *object of beauty*. By adding to the class *producers*, so as to form the higher class *cultivated men*, the complementary class *men of culture who do not produce*, i.e., *intelligent non-producers*, we find the parts of *productive* to be *intelligent* and *executive*. In other words, we find that the attributes making up the character of a man of culture who creates—*productive* in this sense—to be first *intelligence* or *skill* and secondly *executive power*.

§ 136. The class-concept which thus furnishes

one of the parts is called the *principle of partition.* This obviously must always be single for any one partition; but a lower partition may proceed from a new principle.

Partition, like Division, is in logical strictness, properly ever dichotomous. But by successive dichotomies, we may attain a trichotomy or a polytomy.

§ 137. The rules of Partition accordingly are:—

1. Determine the one principle of partition, which is to be found in a class-concept higher than that to which the given attribute belongs.

2. Proceed by dichotomous partition or verify by reduction to it.

3. Enumerate all the parts.

4. Enumerate them in order.

See Appendix, No. XII.—Exercises in the Method of Concepts.

CHAPTER IV.

METHOD OF THOUGHT—THE REASONING

§ 138. As in the case of the Concept, so of the Reasoning, both being derivatives of thought, the *datum* can be verified as real or true to the external object only through the primitive—the judgment. Thus in the sophism: *To do evil that good may come is wrong; to punish is to do evil that good may come; therefore to punish is wrong*, the fallacy can be detected only by verifying the sumption—by determining whether it is true actually that to do evil that good may come in the sense of the proposition is always wrong. Thus also the reasoning: *Fishes are cold-blooded; the whale is a fish; therefore the whale is cold-blooded*, is valid if the subsumption be accepted as true; but that being false, the conclusion is false. The fallacy in such cases lies in the antecedent or premises—the *datum*—and can be exposed only by scrutinizing that. One thing to be done therefore in verifying a reasoning is to verify the *datum*—to recognize it as true to the reality.

§ 139. The order of procedure in verifying a reasoning is the reverse of that in forming it. In the latter case we begin with the *datum*—the an-

tecedent—and from that proceed to the new truth to be found in the conclusion. In verifying, the conclusion being given, we begin more naturally with that and reverse the steps which have been taken in forming it. For purposes of convenience, the method in verifying will here for the most part be given, omitting the method in forming the reasoning as unnecessary.

§ 140. The Objective law of a reasoning respects the logical character of the antecedent. It requires as a condition of perfect thought three things.

First, it requires that the antecedent be recognized as to the number of judgments of which it consists. The observance of this law becomes particularly imperative from the fact that in common discourse, one judgment, as in the enthymeme, is often suppressed.

Secondly, it requires that the antecedent be recognized as to the kind of the several judgments of which it consists, whether affirmative or negative, or disjunctive; whether assertory, problematic, or necessary; whether categorical or hypothetical.

Thirdly, it requires that the antecedent be recognized to be correctly represented throughout in the verbal form in which it appears. The two parts of this rule are, 1. That the language be exactly significant of the thought; and 2. That it be thought in the same sense in the several parts of the antecedent. Thus in the

sophism: *No designing person ought to be trusted; engravers are by profession designers; therefore no engravers should be trusted*, the middle term is used in two different senses in the antecedent. It is the fallacy of the *ambiguous middle* so called.

§ 141. The Subjective law of a reasoning respects the derivation of the conclusion from the antecedent. It requires—1. That a compound reasoning be resolved into its constituent reasonings; and 2. That each constituent reasoning be then thought out in accordance with its own particular laws.

The importance of the first of these requirements is seen in the fact that most unsound reasonings arise from the undiscriminated mingling of divers reasonings. As reasonings admit of indefinite combinations in discourse, immediate and mediate reasonings each being joined with every other in the two classes, while the mind is following closely the movement of thought in one form, it is liable to overlook an error or lapse in some other of the combined forms. Thus in the reasoning, *every orderly arranged thing is vertebrate or is not vertebrate; a crystal is an orderly arranged thing; and it is not vertebrate; therefore it is invertebrate*, there is a combination of a disjunctive reasoning with one by transference; the former is sound; the latter is unauthorized (§ 145).

The subjective law of a reasoning, although

always a mere application of one or other of the four fundamental laws of thought, is modified variously in its application to the different forms of reasoning. These applications will hence be presented in order separately.

§ 142. In logical Conversion, it requires that the character of the copula and of the terms be maintained unchanged.

The special liability to error in this reasoning arises from the fact that generally in discourse the quantity of the terms is changed from extensive to comprehensive, and the predicate of the *datum* which is properly of comprehensive quantity is but one of divers attributes belonging to the subject. Thus *all men are mortal*, when converted is strictly *all mortals are human*, *human* being the proper attribute-form corresponding to the subject-word *man*. But this conclusion is false; it is so because the predicate *mortal*, in the original proposition is changed in the conversion from an attribute into a subject of the attributes of which *human* denotes but one, some mortals being animals. In the loose form of expression, the quantity is changed from being *undistributed*, as it is said, to be *distributed;* that is, *mortal* in the *datum man is mortal*, is not distributed to all of the class to which this attribute belongs; but in the converse *mortals are men* it is distributed, as the subject here includes all individuals having the attribute *mortal*.

The rule of method in verifying a reasoning in

which conversion occurs, that is wherever the terms are transposed, accordingly is:—

Recognize the quality of the copula and the quantity of the terms as unchanged; or if changed, as legitimately changed by some other valid movement of thought, as Restriction, Transference, or the like.

Under this general law, requiring either that the identification of the original proposition be maintained unchanged, or, if modified, be modified by clear logical authority, it is easy to tabulate the legitimate logical conversions allowable to the different classes of propositions determined by combined quality and quantity (§ 86).

The Universal Affirmative (A) is converted by reducing the quantity of the predicate *per accidens;* as *A is B;* therefore *Some B is A;*

The Particular Affirmative (E) *simply;* as *Some A is B;* therefore, *Some B is A;*

The Universal Negative (I), simply; as *No A is B;* therefore, *No B is A;* for if any *B* were *A*, then some *A* would be some *B*, which is contrary to the supposition.

The Particular Negative (O), *Some A is not B*, allows no consequent.

The Single Proposition, *Venus is bright*, is inconvertible, for an individual cannot be predicated.

§ 143. In logical Restriction, the law of method is, if it be Modal, that is, if it be in the copula, that the derivation ever be in the order of the

necessary, the assertory, and the problematical, never the reverse; and if it be Quantitative, that is, if it be in the terms, that the derivation be always from the greater quantity to the less.

§ 144. In logical Disjunction, the law of method requires that the members be recognized as truly disjunct (§§ 36, 37). Thus the reasoning, *angles are right or spherical; therefore no right angle is spherical*, is unsound because *right* and *spherical* are not logically disjunct or opposite, but congruent, for they may both be predicated of the same subject—of the same angle. In like manner, the reasoning, *angles are right or acute; the angle B is not right; therefore it is acute*, is fallacious, for *right* and *acute* are not full complementaries.

If there be more than two disjunct members, they are to be reduced to a true dichotomy (§ 119).

§ 145. In logical Transference of quality from the copula to the terms as in the formula *A is not B; therefore A is non-B*, the subjective law of method requires that the subject be recognized as belonging to the same class as the predicate to which the quality is transferred. Thus the reasoning, *A stone is not vertebrate; therefore it is invertebrate* is invalid, for a *stone* does not belong to the class made up of vertebrates and invertebrates. In the same way, the proposition, *the flying saurian is not struthious, therefore it is carinate*, is valid when it is recognized that flying

saurians, being birds, must be of one of the two classes, struthious and carinate, into which all birds are divided.

§ 146. In logical Composition, the law of method is the same as that of the concept (§§ 126–129).

§ 147. In all mediate reasoning the first step in verifying is to determine whether the derivation is deductive or inductive; that is, whether it is from a whole to a part, or from a part to its complementary part.

Thus in the reasoning: *Virtue is voluntary because it is free, therefore vice is voluntary*, the verification cannot proceed except as it recognizes the movement of thought as inductive—only as *vice* is recognized as complementary of *virtue*.

§ 148. The law of method in verifying a Deductive reasoning directs:—

First, that the conclusion be recognized in regard both to its quality and modality, and also to its terms, the subject being the minor term and the predicate the major term in every valid reasoning.

Secondly, that the other or middle term be recognized to be the subject of the sumption, and also to contain the minor and be contained in the major term, and to be the predicate of the subsumption.

Thirdly, that the subsumption be recognized to be affirmative in quality.

Fourthly, that the quality of the conclusion be recognized to be the same as that of the sumption.

Fifthly, that the modality of the conclusion be no higher than that of the subsumption.

The practical difficulty in verifying this derivation of thought arises from some defective or complicated or irregular form of the statement or from the obscure or equivocal use of language. It is frequently defective by the omission of one or the other of the premises, as in the enthymeme; this defect it may be necessary for a clear recognition of the validity of the reasoning to supply. It is often complicated with immediate reasonings and also with other mediate reasonings, as in the epichirema and the sorites; this complication must be resolved and each reasoning verified by itself. It is often irregular in form by the transposition of the subject and predicate for rhetorical or other purposes, and by the change of the formal order of the three judgments in the reasoning. For practical convenience in verifying this movement of thought the following rules may be of service.

§ 149. PRACTICAL RULES FOR VERIFYING A DEDUCTION.—1. Construct a regular syllogism by finding the proper subject of the conclusion which will be the minor term and consequently the subject of the subsumption; then completing the subsumption by ascertaining the proper middle term in one of the premises, making it affirmative; lastly, forming the sumption with this

middle term as its subject and the predicate of the conclusion as its predicate.

2. Verify the *datum* both in the sumption and the subsumption.

3. See that the middle term is a true middle, in whatever form of language expressed, being a logical part of the major and a whole of the minor term.

4. See that the quality of the conclusion is the same as that of the sumption and its modality of no higher degree than that of either of the premises.

The application of the law may be exemplified in the following reasonings:—1. *His imbecility of character might have been inferred from his proneness to favorites; for all weak princes have this failing.* The conclusion in this reasoning being *This prince was weak*, its subject *this prince* is the minor term and its predicate *weak* is the major term. The sumption then must be *Princes who are prone to favoritism are weak;* the subsumption, *This prince was prone to favoritism.* The subjective law of the reasoning is thus proved to be observed and the *datum* being assumed to be true, the conclusion is valid.

2. *King Louis XIV. ought to be held in honor, for the Divine law commands us to honor kings.* The subject of the conclusion here is *Louis XIV.*, and is the minor term of the reasoning; the predicate, *deserves honor*, is the major term; consequently the sumption must be *Kings deserve*

honor; the subsumption, *Louis XIV. was a king.* But we observe that the word *king*, here representing the middle term, is used in different senses in the two premises; it means in the sumption *Kings as kings*, kings *in their official character;* in the subsumption it means *kings as individuals and in their personal character.* There is therefore properly no middle term and consequently no valid reasoning. This is an example of the fallacy called *ambiguous middle;* or as there are really four terms, of the fallacy called the *Logical Quadruped—Quaternio terminorum*, and of the species called *a secundum quid ad dictum simpliciter*, or a reasoning from an accidental attribute or relation to an essential attribute or property.

3. *The Cretans were liars; therefore Epimenides, himself a Cretan, who gave this testimony, was a liar.* The conclusion gives as the minor term, *Epimenides*, and the major term, *liar;* the middle term with which these are respectively compared is *Cretan;* hence the sumption is *The Cretans were liars;* but the meaning is not that *all Cretans were liars;* consequently *Cretans* is not a proper middle term. This species of fallacy is technically called the fallacy of *unreal universality.*

4. *Mathematical studies improve the reasoning powers; but as logic is not a mathematical study, it may be inferred that it does not improve the reasoning powers.* The subject of the conclusion,

logic, being the minor term, and the term with which it is compared, *mathematical study*, being the middle term, the subsumption is negative—*logic is not a mathematical study;* hence, according to the rule, there is no valid conclusion (§ 85).

§ 150. The law of method in verifying the Inductive syllogism will be more fully stated in the following chapter (§ 180). It is sufficient to say here that it might be laid down in much the same form as in a Deduction (§ 149), except that in the third rule the middle term should be recognized as a true logical complementary of the minor term constituting the same class.

Thus in the reasoning—an enthymeme—*Vice is voluntary for virtue is voluntary*, we begin with the conclusion *vice is voluntary*, and recognizing *vice* as the minor term and *voluntary* as the major term, we construct the syllogism: sumption, *virtue is voluntary;* and observing that the middle *virtue* is complementary of the minor *vice*, and therefore recognizing the reasoning as inductive, we have the subsumption *vice is complementary of virtue;* and conclusion, *therefore vice is voluntary.*

Again, in the induction *the bat is a mammal, for it is a complementary of the monkey and the mole, etc., in being viviparous.* In this enthymeme the sumption obviously is omitted. We construct the regular syllogism thus: *The monkey, the mole, etc., are mammals; the bat is complementary of monkey, etc.; therefore the bat is a mammal.*

Further in the reasoning, *the bat is oviparous; for it is complementary of bird, in being winged;* the regular syllogistic form would be: *The bird is oviparous; the bat is complementary of bird; therefore the bat is oviparous.* The fallacy lies in recognizing the concept *winged* as part of the concept *oviparous*, whereas this attribute belongs not to the class *oviparous* but only to a part of it. In other words the subsumption is false in affirming that *bat*, simply because it happens to be *winged*, is complementary of *bird* in forming the class *oviparous.*

§ 151. The law of method in verifying a Hypothetical Reasoning directs:—

1. That the judgments which form the terms of the sumption be severally verified and that the antecedent member of the hypothetical premise be recognized as involving the consequent either as a whole containing the consequent as a part, or as a part implying the consequent as a complementary part.

2. That the subsumption either affirm the antecedent—the Ponent form—or deny the consequent—the Tollent form ; and

3. That the conclusion affirm the consequent in the Ponent form and deny the antecedent in the Tollent form.

It will be observed that a Hypothetical reasoning can always be reduced either to a Deductive or an Inductive reasoning ; and in verifying, it

will generally be found most convenient to make this reduction.

§ 152. PRACTICAL RULES FOR VERIFYING A HYPOTHETICAL REASONING.

1. Put the reasoning into the complete, regular, syllogistic form; the hypothetical proposition always being the sumption; the affirmation of its antecedent member or the denial of its consequent member always being the subsumption;

2. Verify the sumption by recognizing the consequent member as involved in the antecedent either by some immediate reasoning or by legitimate deduction or induction;

3. Verify the subsumption; and

4. Recognize the conclusion as affirming the consequent member of the sumption in the Ponent form or denying the antecedent member in the Tollent form.

EXAMPLES: 1. *Vice must be voluntary if virtue is; and undeniably virtue is voluntary.* The sumption is: *If virtue is voluntary, vice must be;* the subsumption, *But virtue is voluntary;* the conclusion, *Therefore vice is voluntary.* The sumption is verified by recognizing the consequent *vice must be voluntary* as the inductive inference from the antecedent, being its complementary in the class *moral actions;* the subsumption is recognized as true and as affirming the antecedent. The conclusion, being recognized as affirming the consequent, is seen to be true.

2. *If Logic presents a multitude of facts useful*

to be known, it deserves to be cultivated; but it does not; therefore it ought not to be cultivated. The reasoning being in full and regular form, we proceed to verify the sumption which we find to be true. The subsumption, however, we find denies the antecedent. There can therefore be no legitimate conclusion from this reasoning (§ 94). The conclusion may be true or may be false; this hypothetical reasoning proves neither—proves nothing.

§ 153. The Dilemma is a Hypothetical reasoning in which the sumption rests upon a disjunction, which is established or removed in the subsumption. The reasoning is properly hypothetical; that is, the derivation of the conclusion characteristically turns upon the relation between the hypothetical or antecedent member, not on the disjunction. It is to be verified in the same way as the simple hypothetical; but more care is necessary in verifying the disjunction.

EXAMPLE: *If this man were wise he would not speak irreverently of Scripture in jest; and if he were good he would not do this in earnest; but he does it either in jest or in earnest; therefore he is either not wise or not good.* The reasoning is in regular form; the subsumption affirms the antecedent in respect of both its disjunct parts. The conclusion therefore affirms the consequent alternatively in its disjunct parts.

§ 154. In the Disjunctive Reasoning, the law of method requires:—

1. That the disjunction in the sumption be verified, reducing contrary opposition to pure contradictory when necessary;

2. That the subsumption remove the disjunction by positing or by sublating one of the disjunct members of the sumption; and

3. That the conclusion be of opposite quality to that of the subsumption and have for its predicate the disjunct member not posited or sublated in the subsumption.

§ 155. PRACTICAL RULES FOR VERIFYING A DISJUNCTIVE REASONING:

1. Put the reasoning into the full regular form of a disjunctive syllogism, taking for the sumption the disjunctive proposition, and for the subsumption the proposition which posits or sublates one of the disjunct members;

2. Verify the sumption as a true disjunction, reducing, if necessary, contrary to pure contradictory opposition;

3. Verify the subsumption as a true positing or sublating; and

4. Recognize the conclusion as a positing of that disjunct member of the sumption which is not sublated in the subsumption or a sublating of that member which is not posited in the subsumption.

EXAMPLE: *A fungus is a plant, for it is either a plant or an animal, and it is not an animal.* The sumption here is: *A fungus is either a plant or an animal;* the subsumption, *but it is not an*

animal; the conclusion, *therefore it is a plant.* The sumption is verified by recognizing *plant or animal* as the only kinds of substances to which a fungus can belong—*inorganic being* being excluded if necessary by recourse to the regular dichotomous process. The subsumption being recognized as true, inasmuch as it sublates the disjunct member, *animal*, the conclusion correctly posits the other disjunct member, *plant.*

§ 156. The law of method requires that the compound reasoning, the polysyllogism, whether the *sorites* or the *epichirema*, be verified by reduction to its parts and successive verification of each.

It also requires that the defective reasoning be filled out to its complete syllogistic form.

It further requires that their irregular form, whether the irregularity lie in the changed position of the terms or the changed order of the constituent judgments, be changed for verification, if necessary, to the proper regular form belonging to each variety of reasoning.

§ 157. An unsound form of thought of whatever kind is termed a Fallacy. Thus there are *fallacious judgments*, *fallacious concepts*, and *fallacious reasonings.* But inasmuch as the Aristotelian logic confined itself to the reasoning, the term *fallacy* has more commonly been applied to an unsound syllogism.

Fallacies have been distributed into two classes

of 1. Intentional, called *Sophisms;* and 2. Unintentional, called *Paralogisms.*

Logical Fallacies are further conveniently subdivided into 1. Those which violate the Objective Law, sometimes called *Material;* 2. Those which violate the Subjective Law, called *Formal;* and 3. Those which violate the Verbal Law, called *Verbal.*

§ 158. Of Objective Fallacies, called also Material in the narrower sense, the varieties that have been more common and hence more prominently recognized in logic are:—

1. *Those of an unreal universality;* as in the sophism: *The Cretans are liars; Epimenides was a Cretan; therefore Epimenides was a liar.* The term *Cretans* in the sumption, if taken universally, is unreal. The proposition is true only as meaning that Cretans are characterized for untruthfulness,—that Cretans generally are untruthful.

2. *Those of an unreal middle;* as *That which moves another body is animated; the magnet moves another body; therefore the magnet is animated.* The middle term *that which moves another body* is unreal in the sense involved in the reasoning, as moving *from its own determination.*

Under this general class of Objective Fallacies a number of varieties have been recognized. One is the fallacy *post hoc, ergo propter hoc,* as in the enthymeme: *There will be some great calamity, for a comet has appeared.* The full syllogism

would be: The appearance of comets is followed by calamity; there is now a comet; therefore there will be calamity. The sumption is not true in the sense of the reasoning, for it implies the connection of cause and effect between comets and calamities, and hence a universal and necessary connection.

The fallacy of *non causa pro causa* is but another form of the same vicious reasoning. It alleges some other than the real cause of an event; as "Here was preaching against covetousness all the last year, and the next summer followed rebellion; *Ergo*, preaching against covetousness was the cause of the rebellion." Another example is Bishop Latimer's overthrow of this fallacious reasoning: "Before that Tenterton steeple was in building there was no manner of speaking of any flats or sounds that stopped the haven; and therefore Tenterton-steeple is the cause of the destroying and decay of Sandwich haven. And so," adds the Bishop, "to my purpose, is preaching of God's word the cause of rebellion, as Tenterton-steeple was cause that Sandwich haven was destroyed."

A second is the fallacy of *begging the question*, *Petitio Principii*. Of this species there are divers varieties: 1. The *petitio principii* proper in which a premise is unwarrantably assumed; as the proof that men are bound to obey civil government because of an alleged social compact; 2. The *Hysteron Proteron*, in which the truth of the

antecedent is dependent on that of the consequent; and 3. The *Circle*, in which the conclusion is already contained in a disguised form in one of the premises; as *opium will cause sleep, for it has a soporific quality.*

A third is the fallacy called *Ignoratio Elenchi*, or *Irrelevant Conclusion;* in which the reasoning is correct, but the legitimate conclusion is different from that which was proposed as the proposition to be proved.

§ 159. Of the class of Formal Fallacies, several species have been named. One is the fallacy of four terms, called the *Logical Quadruped*, also *Quaternio terminorum.* Its more common form is where the word which should express the middle term is used in two different senses in the different propositions.

A second is that of the *Undistributed Middle*, in which the middle term is not used in its whole extension in at least one of the premises; as *An Athenian is a Grecian; a Spartan is a Grecian; therefore a Spartan is an Athenian.*

A third is the fallacy of *Illicit Process* of the major or minor term, in which the term is used in a wider sense in the conclusion than in the premises.

A fourth is the fallacy of *Negative Premises*, violating the rule that the subsumption must be affirmative.

§ 160. Of Verbal Fallacies, or of fallacies *in dictione*, six have been enumerated:—

1. Fallacy of Equivocation, in which the same term is used in different senses;

2. Fallacy of Amphibology, in which a phrase or clause is used in different senses;

3. Fallacy of Composition, in which what is true of individuals is asserted of the whole class, as when we infer that because each person in a nation will die in a century the whole nation will perish in that time;

4. Fallacy of Division, in which what is true of the whole taken as a whole is asserted of each part; as *The properly elected legislators have right to make laws; A. B. is a legally elected legislator; therefore he has right to make laws;*

5. Fallacy of Accent, in which by varying the accent or emphasis a different sense is given to a term;

6. Fallacy of Figure of Speech, in which a figurative expression is construed literally.

See Appendix, No. XIII.—Exercises in Method of Reasonings.

CHAPTER V.

METHOD OF THOUGHT.—INDUCTION IN THE DISCOVERY OF TRUTH.

§ 161. ELEMENTARY LOGIC, as has been exemplified in the preceding chapters, exhibits the general doctrine of method as *prescribing the conditions of right thinking in order to its end* (§ 103). These conditions have been recognized as threefold:—Material, Formal, and Verbal, and so determining the threefold laws of method—Objective, Subjective, and Verbal (§ 104). But a twofold procedure was indicated in thought according as the aim or end of thinking is for producing new truth or for verifying a product of thought (§ 105). It was, however, observed that for the most part it is not needful in an elementary treatise to consider separately the modifications in method suitable to these respective ends in thinking. One exception was made to this general remark, however, in regard to logical induction as instrumental in the discovery of truth. The reasons for making this exception and for giving an entire chapter to this one movement of thought are:

First, that logical systems have for the most part neglected this movement and have given but

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imperfect or altogether erroneous statements as to its nature and uses. In their attempt to define it, Sir William Hamilton says "they have been guilty of the grossest blunders."

Secondly, the inductive movement is second in importance to no movement of thought with which it can be co-ordinated. It is one of the most common and familiar in common life, and the grand, characteristic instrument of thought in the advancement of modern science.

Thirdly, an exposition of induction is indispensable to a full logical system or complete science of thought.

These reasons seem to justify the disproportionately extended consideration which is here given to method in induction with the specific end of discovering truth.

§ 162. In the way of introducing the subject of the chapter, let it be remarked that, for the most part certainly, the endeavor to discover new truth—to originate knowledge—is prompted, sustained, and guided to successful results by the inductive principle of intelligence—the inductive instinct, as it may be called. A new discovery may be the result of the merest accident, for which no credit is due to the superior reason of the discoverer, or even to his active reason at all. A tyro in mathematical science may make certain changes in the equation of the ellipse, for instance, in mere amusement, with no rational end in view, certainly with no aim at new truth, and

chancing to interpret some new form of the equation may find it to express a truth respecting this geometrical figure of which he had never dreamed. The most casual glance at any phenomenon in the world of change about us, or at any substance, may bring to the intelligence a new cause, a new effect, a new property. But proper rational discovery, seeking with express intent that which is new, can hardly be supposed to be prompted and guided otherwise than by some anticipation, more or less conjectural, arising from what one already knows:—from this he starts to find what may be like it. Sir Isaac Newton, the story runs, is led to the discovery of the universal law of gravitation by the notice of the fall of an apple to the ground. Mr. Charles Goodyear discovers the useful process of vulcanizing caoutchouc from noticing the effect of intense heat on a piece of rubber which he has carelessly thrown into a stove of burning anthracite coal. More recently, Mr. Tyndall observes the absorption of light, first from incandescent lime, then from incandescent platinum, now through a long tube, then through a short tube, now filled with vapor, then with the vapor condensed to a liquid, and so attains his law of "thermal continuity." These are but exemplifications of the general fact that discovery, origination of truth, is prompted by the instinct of knowledge under the guidance of the inductive principle. That like supposes like is a principle which may obviously lead to what is

utterly unknown as well as to that which is anticipated. That a given cause produces one effect awakens the just anticipation that it may produce some other effect hitherto not known or dreamed of. Experiment or search prompted by this form of the inductive instinct may lead, often in fact does lead, to the discovery of the new effect.

Often a more general or a more comprehensive truth is known, and the inductive movement will terminate in attaining some particular truth or fact. The planet Uranus is found to suffer some disturbance in its motion; and as such planetary disturbance in other cases is from some member of the solar system, an inductive conjectural anticipation leads to the discovery that Uranus is not a star but a planet; what is known only as a disturbing cause generally, is ascertained specifically to be a planet.

More commonly in discovery or origination of truth there is a conjectural anticipation of what may be found to be. Such conjectural truth may be verified just as one that is already known and is simply to be demonstrated. The inductive procedures in the two cases will of course be identical. There need be consequently no formal exposition of the particular modifications of conditions applicable respectively to them. We forego any further consideration of what may properly be called the *telic* conditions of induction—the conditions specifically prescribed by the

aim or end in thinking, as not required in a rudimentary treatise. The remarks just made under this head indicate the importance to the most effective and successful prosecution of scientific discovery, whether the aim be to discover laws or facts, new substances or new properties, new causes or new effects, of a thorough and familiar acquaintance with the nature, the principles, and the processes of induction. The culture of the inductive instinct is a fundamental requisite in order to the highest success to the seeker for new truth.

§ 163. Of the other conditions of perfect thinking, the first class named, the *material* conditions, may be summarily comprehended in the statement that the *datum* must be a true cognition. Otherwise, however perfect the thinking exerted upon the matter of thought, the conclusion must be illusive. Real truth can be obtained only on the condition of the true apprehension of the object to which the thought is applied. In order to secure compliance with this condition, it will be found helpful to acquire a ready habit in discriminating the different sources and avenues by which the object of thought is presented to it. These sources are generally twofold: (1) thought itself; (2) observation. The legitimate fruits of true thought can never be questioned. They constitute the one highest class of truths—*necessary* truths. The sources of proper observation are either (1) *Intuition*, the

source of supersensible truths; or (2) *Perception*, the source of those truths which are presented to the mind through the outward sense.

It is hardly necessary to remark that, in whichever department of truth as determined by these several sources or avenues of thought the inductive movement proceeds, care should be taken that the *datum* is fairly attained in conformity with the laws which govern the apprehension of truth from those different sources.

The *verbal* conditions in induction are substantially those which are required in all thought. At least the special modifications made in their application are too obvious to require any special consideration of them (§§ 111–113).

THE FORMAL CONDITIONS OF PERFECT THOUGHT IN INDUCTION.

§ 164. This class of conditions in inductive thought requires for our purpose a full and detailed exposition.

As induction enters so largely and so freely into thought generally in combination with other forms of thought, sometimes predominant giving character to the whole complex movement, sometimes only incidental and auxiliary, sometimes prominent and explicit, sometimes implied and obscured although perhaps governing and pivotal to the reasoning, the first thing to be attended to is to recognize distinctly the inductive

part that is proposed in the movement, and its relationship to the rest of the thought, whether it is to be used as principal or as dependent; and also to discriminate it carefully and accurately from all other processes of attaining knowledge, particularly, from all other logical processes. It is thus, first, to be distinguished from all logical movements terminating with mere concepts. It is, as we have seen, essentially and characteristically a reasoning—a movement having to do with judgments, beginning with a logical judgment, mediated through a term by a judgment, and terminating with a judgment. New truth may be attained and science may be advanced by simple changes in concepts; and this in a twofold way: —(1) by *Synthesis* or amplification, which may be either by generalization or the amplification of a subject-concept, that is, of a term in its extensive quantity, or by determination, or the amplification of an attribute-concept, that is, of a term in its comprehensive quantity. Science is advanced thus by discovering and adding a new asteroid to the class of planetary bodies;—the genus *planet* is amplified by this synthesis; in other words, the generalizing process is carried one degree farther and more truth is attained. In an analogous way, new truth is gained by amplifying the comprehensive quantity, that is, by logical determination, when a new attribute is discovered and added to the aggregate of attributes common to the genus, as by the discovery

of the mineral constitution of planetary bodies through the spectroscope.

(2.) Science may be advanced by Analysis: the reduction of concepts either by logical Division,—the reduction of a subject-concept, that is, of a term in respect to its extensive quantity; or by logical Partition,—the reduction of an attribute-concept, that is, of a term in respect to its comprehensive quantity. Thus, by Division, *planets* are distinguished into Primary and Secondary. By logical partition, they are known as gravitating, revolving about the sun, etc., the comprehensive attribute *planetary* being reduced to these more particular constituent attributes. Knowledge is advanced in either way, as a genus is resolved into smaller groups (= species) or a comprehensive attribute into its constituent characters.

This movement of thought, consisting merely in a change in the quantity of concepts, is a most familiar one. It may precede or attend or follow some other movement, as induction or deduction or other reasoning, or original observation. Because very frequently following induction in scientific study, generalization has by some been confounded with it. The error is exemplified in the case of those who define or rather describe induction as the process by which general facts or truths are attained.

In the next place, induction is to be discriminated from all immediate reasonings. Even by

logicians it has been confounded with that form of immediate reasoning called composition. This is indeed an error but little differing from the one just noticed; since logical composition is but the expansion of logical amplification into the form of a reasoning. Thus Sir William Hamilton exemplifies the inductive movement in this way:—"This, that, and the other magnet attract iron; but this, that, and the other magnet represent all magnets; therefore all magnets attract iron." The movement here is simply this: Every individual magnet attracts iron; therefore all magnets attract iron. There is no true mediate reasoning here; only an utterly illusive form, an empty mask put upon a different movement of thought. There is no mediation of one judgment through another. It is a mere synthesis of the individual constituents into a whole taken distinctively.

Once more, induction is to be discriminated from deduction. These two movements, as has been before noticed, constitute the class of thought-forms called mediate categorical reasonings. The two are consequently complementaries of each other, making up the whole of that class. And they differ specifically just in this one particular: deduction moves in the one of the two comprehensive relationships of logical quantity —that of whole to parts; and induction in the other of these complementary relationships—that of part to part. The discrimination is

natural and even necessary in any full analysis of thought-movement; it is withal so familiar that it would seem it could not be mistaken or overlooked. But logicians appear to have stumbled over its very simplicity. Recognizing the fact that there is something called induction, and looking far and away beyond, they have unreflectingly alleged that it is the reverse of deduction, moving backward and upward from part to whole, as deduction moves forward and downward from whole to part; as if mere direction of movement, progressive or retrogressive, could support so wide a distinction as that between deduction and induction. This conception of induction, moreover, overlooks entirely the one great relationship necessarily given in any analysis of logical quantity, and utterly subverts the accepted doctrine that deduction rests upon previous induction. Other logicians have maintained that all reasonings, all syllogisms, are deductive; and have striven, but vainly, to transform the inductive reasoning into the form of the deductive. They err too from overlooking this quantitative relationship between part and part seemingly because of its extreme simplicity and familiarity. The truth, it would seem, is beyond all question and must be recognized on the most cursory reflection. Deduction moves characteristically in the relationship of whole to part; induction in the coördinate relationship of part to part; and

the laws that govern in the respective movements must in all reason differ in essential respects. Particularly, we must not suppose a reasoning to be necessarily fallacious, simply because we fail to discover any proper deductive movement in it.

When the inductive movement in a complex process of thought has been identified and carefully discriminated from all other forms of thought, the next step will be to see that the formal conditions of a true and legitimate induction are observed. These conditions are directly deducible from the essential nature of the process as before expounded. From the very definition of the process, we may here repeat, it will appear thus that every legitimate induction, when developed into its full and normal form, must embrace three judgments or propositions,—the two premises, major and minor, otherwise denominated sumption and subsumption, and the conclusion. Of these, the sumption must identify or difference a term as part in relation to another term which is a whole to that part; the subsumption must identify this part with another like or complementary part; and the conclusion must identify or difference this complementary part with the whole, according as the sumption has identified or has differenced. As being complementaries the two parts must be given by an analysis of the whole proceeding on the same principle—that is, by an analysis

through the same attribute, if the terms are taken in their extensive quantity and through the same subject, if taken in their comprehensive quantity. It is not necessary in order to the legitimacy of the movement that the parts should be of the same rank or degree in the scale of division or partition, since a part of a part is logically a part of the whole. Thus any one constituent of an aggregate of properties is as truly an attribute of any subject as the entire aggregate; *man* is as truly *intelligent* as he is *rational*, which includes *intelligent*. So an attribute belongs as truly to each individual of a species as to the species; *Socrates* is as truly *rational* as is *man* of which species he is a part. In such cases where the reasoning thus overrides the distinction of rank or degree in the analysis or synthesis of a term, strictly speaking, we have a complex movement—a kind of epichirema—which, in rare cases for verification, it may be necessary to resolve. Ordinarily this is unnecessary; and we need not cumber a thought with a distinct consideration of it.

In these three propositions there are accordingly three terms, and, if the induction be single, only three terms;—one being a whole of which whole the other two terms are complementary parts. The whole and one of these two parts must be given; they are the terms that appear in the sumption.

Practically the inductive movement is more

commonly in the form of an enthymeme, one of the premises being suppressed. If the induction be legitimate, the reasoning may be developed into the full form of a regular inductive syllogism; and in order to an assured verification it may be necessary to give it this full development. If the enthymeme consist of the sumption and the conclusion, then the subsumption which affirms the minor and the middle terms to be complementaries, must be supplied: as in the familiar case of the child that after one experience of burning his finger shuns a second exposure to the candle flame. He practically reasons: that finger was burned by the flame; therefore this other finger will be. He suppresses the subsumption that the fingers are like each other. So the botanist reasons: this buttercup has five petals; any other must have five. The reasoning in full would be: this butter-cup is five-petaled, P is W; that other is a complementary of this, the two being like parts of the same kind of flower, C is P; therefore, it is also five-petaled, C is W. He might suppress the sumption thus: this flower is the same as that—a buttercup, C is P; therefore, it is five-petaled, C is W. The sumption to be supplied would be: this flower is five-petaled, P is W.

It is apparent that the peculiar difficulties attending the inductive process are these two;—(1) that of determining whether the given part is, as assumed, a part of the given whole; and (2)

whether the two parts are true complementaries. But wholes are diversely constituted; they may be made up of many different kinds of parts; and a part of one kind will not of course be a proper logical complementary of a different kind of part of the same whole—will not be a like part. It is hence obvious that an induction which should partially move in one kind of whole and partially in another, without the modification of the procedure which the character of the particular kind of whole concerned at the time may require, will be blind, stumbling, and illusive. It is an obvious first principle in all legitimate induction that the procedure continue ever in the same kind of whole, or at least if there be a change, that the procedure modify itself accordingly in distinct recognition of the changed character of the whole. Moreover, it is clear that inductive inquiry will be greatly aided by a familiar acquaintance with the different generic classes of wholes in which such inductive inquiry on the part of the human mind may move. Science has been defined to be generalized knowledge. The definition is inadequate; for, not to criticise it farther, there is science, as we have seen, which is attained by logical determination as well as by generalization, in comprehension as well as in extension. But in each case science respects a whole and is consummated in a whole; and the procedure in attaining it is through one or more of the different kinds of

parts of which the whole is made up. The different particular sciences are distinguished one from another by this specific character of the whole which they severally respect. Of eminent service it must be to every scientific explorer to be able, on intelligent, rational grounds, at once to recognize the specific character of the whole in which his research is to move, as it is to the geographical explorer to have the different districts in the country which he proposes to explore already outlined in his mind; to know whether his explorations are to the north or to the south, to the east or to the west. It is irrational to rely on blind instinct when clear intelligence is attainable. The pursuit of truth will in this rational intelligence be more confident as well as more easy.

In order to facility as also to accuracy in inductive investigation, an exhaustive enumeration of the different generic kinds of wholes in which induction may move will now be given, with exemplifications of the inductive process in each, with also an indication of some of the fallacies to which they are respectively liable.

CLASSIFICATION OF INDUCTIVE WHOLES.

§ 165. All the wholes with which rational thought can be conversant are either,

I. LOGICAL WHOLES—wholes which are products of thought itself; or

II. REAL WHOLES—wholes in actual being.

§ 166. I. *Logical wholes* or the wholes of pure thought. These embrace two species: (1) *Generic wholes*, or composite subject-wholes—class-wholes;—and

(2) *Comprehensive wholes*, or composite attribute-wholes.

A generic or class-whole is exactly circumscribed by the attribute common to all the constituents of the class. This attribute is denominated the *base* of the generic concept (§ 56). An induction when dealing with this kind of whole should therefore proceed ever in clear recognition of this base of the concept—this attribute by which the class is determined.

The liabilities to error here are diverse:—(1) The attribute on which the class is formed may be erroneously assumed. A class of bodies was once erroneously assumed as light or non-gravitating, on the ground that they possessed the character or quality of levity. An induction here would of course be as illusive as the assumption is groundless.

(2) The constituents of the class may have been erroneously assumed to possess the common or class-attribute. A bright body was not long ago observed between Mercury and the Sun. One astronomer referred it to the class of planets, while another referred it to the class of stars. An induction is supposable in either class, the planetary or the stellar; and might have led in

the one case to the conclusion that the body revolved about the sun, reflected its light, etc., and in the other case, to the ascription of quite different characteristics. The induction must necessarily have been ambiguous in its result, until the body should be assuredly determined to belong to the one class or the other.

If the verbal conditions of right thinking are observed, one of which is of especial importance here arising from the ambiguity of terms in language as to their being of extensive or of comprehensive quantity, and if, moreover, the class has been formed on a true attribute, and the constituents gathered into the class because possessing such a true attribute in common, the inductive movement must conduct to certain truth. The result must partake of the necessities of thought.

In an analogous way, the method of induction in a whole of comprehensive quantity involves, in order to perfect thought, the necessities, first, that there be a true subject possessing the common attribute ; and, secondly, that the constituents of the attribute all belong to the same subject.

If, for illustration, we assume an attribute whole as belonging to a subject called *centaur*, which attribute whole consists of the constituent attributes, *human-headed* and *horse-bodied*, we may have this induction : In the centaur as an animal whole is *sustenance by adequate food* as a compo-

nent attribute; *mouth-capacity* for taking food is a complementary attribute of this, since *sustenance by food* and *capacity for taking it* are both necessary constituents of animal being; therefore, the human-headed centaur had the mouth-capacity of a horse, that is, by further inference the human mouth is sufficiently capacious for a horse's stomach. Manifold other absurd conclusions might be logically derived from the mature recognition of a subject having this composite attribute—*human-headed* and *horse-bodied.* By the necessities of thought an absurd conclusion proves untrue premises or an illogical procedure. The whole reasoning is a complicated one; but the vital element in it is an induction by which the attributes *human-mouthed* is inferred from an assumed complementary attribute—*horse-capacity of growth.* In the same subject—*centaur*—at the first step is found the composite attribute *human-headed* and *horse-bodied.* These accordingly are complementary attributes in the notion *centaur;* and any essential constituent of one of those attributes is partially complementary of any constituent of the other, so that the presence of the one necessitates the presence of the other.

§ 167. II. Real Wholes, or Wholes in Actual Being.—Real wholes are of two species: (1) *Substantial Wholes*, and (2) *Causal Wholes* (§ 39).

Substantial Wholes are aggregates of qualities constituting individual substances in actual being.

Causal Wholes are the wholes of changes in actual being constituted of their beginnings or *causes* and their results or *effects*.

Every real object in nature may be regarded either as under the relations of substance and quality, or as under those of cause and effect. Even the most inactive of material substances, as a pebble, may be truly thought as a cause, since having at least gravity, it must act by virtue of this gravitating force in attracting other material objects. In strictness of language, indeed, it is only the force or energy in matter which makes it a cause. It is this energy in matter which effects the unity of action and result and makes it a whole—a causal whole. There is inexactness and consequent liability to error when a portion of matter thus pervaded by energy, as a pebble is pervaded by gravitating force, is thought as the cause, and the portion of matter in which is left the result of the acting force, is thought as the effect. The matter, in which the force resides and acts, only defines or indicates the limits of the acting force. It serves to designate the force, but is not the force itself—is not a true cause. Ordinarily the designation may safely be taken to represent the cause; but there is a liability to error which in strictly scientific studies must be carefully guarded against. The beginning of science, as also popular intelligence, deals with objects as substances rather than causes. As it advances it directs its attention

more to the forces or energies that are thought to act in substances and to be manifested through them. It is, however, only through some energy which impresses our senses that we can come to a knowledge of any substance. It is with energy, accordingly, that science in its last reaches will chiefly concern itself. Energy or force in action, as we elsewhere show, is resolved at once by us when we come to think it into a beginning and a resulting state; that is, into cause and effect.

Real wholes, whether substantial or causal, are of two species: (1) *material* or *physical;* and (2) *mental* or *spiritual.*

SUBSTANTIAL WHOLES.

§ 168. The method of induction in the case of a substantial whole is analogous to that in the case of a logical comprehensive whole. The only difference is that the substantial whole is a real object, or if imaginary, is treated in the thought at the time as real, and is thus a real to the thought; the comprehensive whole is one that is constituted by thought alone. We may form wholes by combining this and that and the other quality into an aggregate, and do this indefinitely, constructing wholes that may have no correspondence in actual being. But men form logical wholes mainly for actual uses in dealing with real objects. Especially do they forbear putting into permanent forms of language such mere thought-

products, unless corresponding to real things. Comprehensive wholes and substantial wholes accordingly appear, for the most part at least, in identical forms of language. Generally, perhaps, accepted words in language, although strictly denoting only wholes of thought, may safely be assumed as denoting real wholes in nature, for in the long run, they are allowed in speech only as indorsed by common observation or experience. But there may arise occasions requiring careful inspection, so as to secure a proved correspondence of the thought with the actual object. Here or there an *ignis fatuus* may have been taken to be a real ghost.

In dealing with this kind of whole, accordingly, the material conditions of perfect thought require first, that the *datum* be a real;—that, in other words, it be a substance truly apprehended in actual observation or experiment; and, secondly, that this substance actually contain the qualities which are thought as belonging to it. The substance is the whole in the induction and the qualities are the parts, except in the largest wholes of this class, in which substance and quality are regarded as the complementaries, as will presently be shown.

It is evident now, that in any substance whatever, truly apprehended, if one constituent part be given, any other complementary part may be induced. Thus if the whole be a mass of water, the known constituents of which are oxygen and

hydrogen, if in it be found a certain quantity of oxygen, we induce with certainty that there is also a certain quantity of hydrogen. This is indeed a very simple matter; but it is of the essence of the most complicated procedures in this species of induction. Limestone is a whole constituted of carbonic acid and lime; but carbonic acid is constituted of oxygen and carbon, and lime is constituted of oxygen and calcium. Given then a whole of limestone, if we find in it the metal *calcium* as a constituent part, we may infer that carbon is to be found in it and in its proportionate quantity. From the presence of any remote constituent of a substance, we induce legitimately thus to the presence of any other constituent however remote. The movement of thought involves here the following up of the given remote part to a higher containing part, then passing by simple induction to the like complementary part, and finally passing down from that to the lower contained part. Thus in the example given, finding in a piece of limestone the calcium, as a remote part, we first find the whole—lime—of which it is an immediate constituent; then we induce to its complementary part carbonic acid; and then, finally, we come down to one of its constituents—carbon.

Every material substance may be regarded as constituted of either of two distinct kinds or orders of parts: (1) component parts corresponding to the parts in the extensive quantity of logic,

forming mass-wholes or group-wholes; and (2) qualifying parts, corresponding to the comprehensive parts of a logical whole. The inductive movement is valid alike in each and proceeds in analogous ways, requiring no further distinctive consideration. The general quantitative relationships of parts to wholes, and of parts to parts, furnish the one valid ground for the thought-movement in both cases.

There is a further distinction to be recognized in inductions proceeding in this kind of wholes which is of more importance. There is a four-fold relationship existing in substantial wholes on which are based four different movements of inductive thought.

First, we have the relation of substance to quality. Every substance has its aggregate of qualities; or more exactly, perhaps, *is* its aggregate of qualities. This relationship is of a purely thought origin, and bears the character of necessity which attaches to all legitimate thought. Every substance accordingly is so far identical with each of its constituent qualities, as every whole is so far identical with each of its several parts. Given therefore in thought a substance, we induce to its qualities—it has qualities of some kind; for substance and quality are true complementaries of one logical whole necessarily implying each the other.

Secondly, we have the relation of quality to substance, which is the inverse of that just

named. Given thus a quality, we induce under the necessities of thought to a substance of which it is a constituent part.

Thirdly, we have the relationship of substance to substance, in which an induction may move, when the two are recognized as parts alike of a more comprehensive substance.

Fourthly, we have the relationship of quality to quality, when recognized as complementary parts of the same substance; so that, given a certain quality of a given substance, we may induce to another quality of the same. Thus, finding a certain quantity of nitrogen in a certain volume of atmospheric air, legitimate induction authorizes us to expect its proportional quantity of oxygen.

It is obvious that the inductive thought will move with greater facility and assurance if the particular one of the four movements just indicated be distinctly recognized. This should therefore be carefully determined at the outset. Generally it may be observed that the narrower and the more circumscribed the field, and the more simple and the more definite the movement of the thought in it, the more easily and the more surely will the result be obtained.

KINDS OF SUBSTANTIAL WHOLES.

§ 169. In whichever of these four relationships in substantial wholes the induction may move, it will of course suffer some modification from

the specific nature of the whole. Only the more general classification can be given here. We have thus, as already stated, the two most generic divisions of (1) material and (2) mental or spiritual.

Of material substantial wholes we have the subdivisions of (1) Inorganic and (2) Organic. Of the Inorganic, there are the distinctions of crystalline and non-crystalline; also those of solid, liquid, and gaseous, in each of which we discover peculiar kinds of qualities. As the identities or samenesses which are involved in induction are in substantial wholes qualitative, and the parts are consequently qualities, the special care required here must obviously be directed to the special kind of quality concerned. The selection of the kind of qualitative whole may sometimes be free; sometimes, it may be determined by the nature of the object or by the aim in the induction. In either case, it should be distinctly recognized, as the induction can be valid only in case of like parts, that is, only when the qualities are of the same kind.

It is not necessary that the classification of these wholes should be the most perfect that science can give. The validity of the induction does not depend at all upon that. But the classification is serviceable as helping to narrow the range of the thought, and so to assure accuracy.

In mental or spiritual being, the highest partition is into (1) thought, (2) feeling, and (3) will.

In lower forms of partition we have the respective qualities that make up these several functional attributes, as in the case of thought, *apprehensive* and *discursive;* of feeling, *impressible* and *impressing;* of will, *selective* and *dominant*, and the like.

The fruits of mind, the products of mental activity, so far as respects these qualities, are substantial wholes. Causes, even, and effects, regarded as having qualities, also belong here. It should ever be borne in mind, that the distinctions of cause and substance, of effect and quality, are distinctions created by thought and not the distinctions of the real world. There are not in the real universe two suns, one round, bright, and the like, and another attractive, illuminating, heating, etc. Thought distinguishes for its own uses these two views that may be taken of its object, and demands for its own end of perfect truth, that the two be not confounded. Identity, likeness, which the nature of thought prescribes as its fundamental law, must run in one single track, and must not be capriciously passed from one view to the other.

CAUSAL WHOLES.

§ 170. The method of induction in the case of causal wholes is widely different from that in substantial wholes, as different as is physiological study from anatomical. Here the whole is con-

stituted of an action, or, abstracting from the conception the notion of an agent, is constituted of a motion, having a determinate beginning and end—the beginning motion being designated the cause, and the ending motion, the effect. Generally, however, language designates the beginning motion by the portion of matter in which it takes place, as also the ending motion in the same way, by a piece of matter left in a certain state or form by the motion. We thus familiarly call the gunpowder—that is, the matter in which, or from which, the motion begins—the cause, and the perforated target the effect. Matter itself in stricter thought we recognize as inert; and accordingly, even when using language which literally implies a material cause or a material effect, we in thought suppose a force acting in or through the matter.

As in a substantial whole we are under the necessity of limiting or circumscribing that portion of the continuous extent of matter around us which is to constitute the particular whole of which we propose to think, so in the endless continuity of motion we are under the necessity of cutting out or defining just that portion of the continuous succession with which our thought at the time is to deal. No little care is requisite often in determining with exactness the defining lines of this selected portion of the ever acting force or the continuous motion in the succession of time. The powder and the explosion may

form one causal whole; the powder and the perforated target another; the powder and the won prize a third, and so on. In the three causal wholes mentioned we have first the powder, representing the particular force from which the motion proceeds or which represents to us the particular motion as it begins and so constitutes the cause in the several cases; and then the explosion, the perforation of the target, the winning of the prize, as the three several effects.

As in the case of a substantial whole, again, the two or more complementary parts constituting the entire causal whole may each comprise a plurality of smaller parts, all of which, by virtue of their being respectively constituents of the primary complementary parts, stand in the relation to one another of being complementaries and so allowing and legitimating the proper inductive movement. We say thus in familiar discourse that the sulphur in the gunpowder becoming ignited produced the explosion; or the gunpowder exploding perforated the target. We thus freely induce from any part of a causal whole to any other part standing in this complementary relation. From the sulphur as part of the gunpowder we may induce to the perforated target as part of the effect in the causal whole. The logical principle here being, as before, that the parts of complementaries are complementaries to each other.

There is, moreover, in causal wholes as in sub-

stantial, a four-fold relationship involved, presenting four distinct lines of movement along which inductive thought may run: first, the relation of cause to effect; secondly, that of effect to cause; thirdly, that of cause to cause; and fourthly, that of effect to effect.

In the first two cases, the two complementary parts between which the inductive relation exists are cause and effect, which together constitute one causal whole, the movement of thought being in the one case the inverse of that in the other. In the third case, the two complementary parts are two causes constituting one comprehensive cause, that is, two parts of the same cause. In the fourth case, the two parts are two effects of the same cause, in other words, two like parts of the one comprehensive effect.

The first case is that of an induction from cause to its effect. The movement of thought here is legitimate and valid as from one part to its complementary, since cause and effect which are given to thought in any single real act or motion, are parts into which it is the proper function of reflective thought to resolve it. The relation is a creation of thought, and, as elsewhere observed, bears the character of necessity given by legitimate thought. The idea of cause, thus, is not an innate idea of the human mind; nor is it an original, underived phenomenon, emerging spontaneously and from no grounds; nor is it the product of some mysterious, natural

or supernatural function of the mind other than the intelligence itself. All these notions of the origin of the idea of cause are unphilosophical; they are groundless and needless. Given an act, a motion, single and simple as it may be in itself, and reflective thought at once and by the instincts of its own nature, in perfect analogy with its operations elsewhere, resolves this single act into the beginning condition and the ending condition of the force which is the essence of the act. These two parts, the beginning and the ending, we call respectively cause and effect. The motion is indivisible in outward reality; it is apprehended as one and individual; thought resolves it into a dual—the dual of cause and effect, precisely as it resolves any object apprehended as single into the dual of substance and quality. The distinction does not exist in the external reality; no man ever saw a proper cause existing separately from its effect, side by side with it. As a creation of thought the truth, which the recognition of identities involved in the relation of whatever form originates, is necessary truth—truth which is beyond question, since thought cannot deny itself. The truths, springing thus into being in our minds, in legitimate thought when turned upon the causal relation requires no universal assent, no uniform experiences in order that they may be substantiated. They rest on the nature of thought itself; emphatically here "truth is truth to the end of reckoning"; it

shines by its own light and asks no help of reflections from the outer darkness. Here we find the solid foundation for all science so far as it respects motion, attraction, force, legitimating and validating it, directing and sustaining it in every step of its progress. It necessarily begins with experience, with observation of an action or motion; knowledge, thought, must have an object and that object must be from without the thought—must be given to it in order that it may be apprehended. This action or motion thus given and apprehended, it is the function of thought to resolve, to analyze, and out of the manifold dualisms which it thus attains grows all truth, all science, all knowledge. This whole department of science rests thus on the primitive induction from cause to effect or inversely from effect to cause as complementary parts of one causal whole. All the contingencies here, if the induction be legitimate, arise out of the inaccuracies or the inadequacies of the human mind in attaining the *datum* to thought. This then is the universal law in this case of causal induction:—Given a cause, we induce under the necessities of thought to its effect as its complementary part. The causal whole is the action or motion in which the force acting has a beginning form which we call its cause and also a resulting form which we call its effect. From the cause as one part of the acting force we induce the effect or its complementary. Given, thus, a rising of temperature to

a degree noted as 36° by Fahrenheit's thermometer, we induce from it the melting of ice into water when in suitable condition to be freely acted on by the force concerned. The essential factor here is heat-force recognized as of varying intensities. The force acting does not spend itself and cease to be; it is conserved, although made apparent to us in another form. The nature of that which we call matter and its relations to force or energy, science has not yet definitely determined. It is a subject for physics or for metaphysics, however, not for logic, which must in its illustrations accept the expositions of the nature of real objects that other branches of science or philosophy bring to it.

The second case is that of an induction from effect to its cause. The causal whole here, as in the first case, is cause and its effect as the two thought-parts of a single movement observed in external reality, the one part necessarily implying the other. The general law is: Given an effect we induce as its complementary an operative cause. Ice having melted, we induce as its cause a rise in temperature above 32°.

The third case is that of an induction from cause to cause. Here the two complementary parts are two causes constituting a single larger or more comprehensive cause. The sun thus as a single comprehensive cause works in a twofold way, causing heat and causing light. Given accordingly an increase of heat as causing the rise

of the mercury we induce an increase of light as causing vision of what was before obscure.

The fourth case is that of an induction from effect to effect. The parts here are the two particular effects constituting in whole or in part the whole of some comprehensive effect.

The last two cases obviously have nothing peculiar to them distinguishing them from the most simple and ordinary inductions. The first two, from cause to effect, and its inverse from effect to cause, are the most important forms of induction in causal wholes. The liabilities to error in these causal inductions are many and peculiar; they will be best exposed by a clear discrimination of the particular kind of causal whole in which the induction moves. An enumeration of these kinds with exemplifications of induction will be given as helpful to the securing of accuracy.

KINDS OF CAUSAL WHOLES.

§ 171. Causal wholes are conveniently classified on the principle of the recognized distribution of the forms of energy or force observed in the world around us. It will be borne in mind that the essential thing in causal induction is the energy or force that acts, not the matter in which or through which it acts, or by which it is circumscribed or designated. The specific energies or forces with which we are conversant, then, are the following:

I. *Physical*, embracing mechanical force; molecular and polar force; heat, light, electricity, and magnetism;

II. *Vital*, comprising (1) organic and (2) animal force.

III. *Rational* or personal, comprising (1) instinctive and (2) human, these being the two forms of rational energy with which science has chiefly to do.

§ 172. *Gravity as Cause.*—In those causal wholes in which gravity appears as the force or energy concerned, the cause is more properly designated by the force itself, but with suitable care it may also be designated by the matter in which the force appears and from which it begins its operation in the given whole. Induction may be from cause to effect or from effect to cause. For illustration in one of its simpler forms, observing the gravitating force in this piece of iron, or what is equivalent, recognizing the iron as gravitating, we have a cause; as such cause, it presses down the hand on which it rests. We induce from this that it will, if in like manner applied to it, press down in like manner the other hand—any other hand—any other material substance in like relation to it. We may enlarge our causal whole so as to embrace in it any other form of matter and so induce generally that every other mass of matter will press every mass other than itself. We reach thus the law of universal gravitation: that is, not that the whole mass of matter

gravitates as a single whole, but that distributively each particular mass of matter gravitates. The example illustrates the truth and so the importance of the principle that we reach the universal in inductive inference only through the parts, and the whole to which we conclude is to be taken distributively to each part. But we should deem it rash to rush to so wide an induction as this, that gravity is universal in matter, from a single observation. This however is not because one induction, if in every respect legitimate with a true *datum*, does not give absolute certainty; but simply because we are unable to assure ourselves that all other forces are excluded in the observed fact. The exclusion, effected perhaps under the logical laws of disjunction, carried sufficiently far, might legitimate the conclusion.

We attain in a similar way by induction the general law of gravity that it varies inversely with the distance and that the degree of this inverse variation is determined by the square of the distance. Newton having found on careful investigation that the moon falls towards the earth according to this law, induced that to be true of every other satellite revolving around its primary. In other words: from the moon's motion as effect in the causal whole of gravity, he induced to a like cause operating in the case of every other secondary in the solar system. Had every other cause been excluded, the induc-

tion would have been legitimate and therefore would have approached the nature of a necessary truth. The contingency attending it comes from the observation as possibly inaccurate or from the non-exclusion of every other cause. This contingency obviously would grow less and less as observations with inductions should be multiplied, all leading to uniform results. It accordingly concerns the *datum*, not the integrity of the inductive movement itself as legitimately conducted.

Another illustration of induction in a causal whole of gravitation is found in the familiar history of the discovery of the planet Neptune. The planet Uranus was observed to suffer some disturbance in its motion. Here was an effect to be traced to its cause. The causal whole was defined by the planetary system of matter; the only cause was the force of gravity. By disjunctive exclusion of all other forces induction from the given effect led to a cause outside of the orbit of any other known planetary body and thus to a planet previously unrecognized.

§ 173. *Molecular Force as Cause.*—We may enter anywhere into the field of phenomena comprised within this causal whole and selecting any fact of observation begin a sure system of inductive science. The comprehensive fact here is the combination of different elements into a new chemical body with its own properties. The healthful salt of our tables is the result of the

combination of a poisonous gas with a bright silvery metal. The combination brings out certain properties, entirely foreign to the combining elements and as unlike almost as can be imagined. A single observation of one instance of such combination justifies the induction that the chemical properties of a body formed by a chemical combination may have little or no resemblance to those of either of the substances entering into the combination. A number of observations might be necessary even to awaken the suspicion that elementary substances enter into combination with one another in definite proportions; more might be requisite to justify the conclusion that this is a general fact, for other causes, known or unknown, might need to be excluded. Yet the procedure might be predominantly one of induction from the beginning. One observation that the chemical equivalent of oxygen is *eight* when combining with one part by weight of hydrogen justifies the conclusion, if other causes be excluded, that in all cases of the chemical union of these elements this will be the ratio of combination. A few experiments will suffice to exclude the supposition of other causes operating. And this equivalent thus ascertained in the union of oxygen and hydrogen is the valid ground for the conclusion that in the case of the union of oxygen with any other element this law of equivalents will be observed. So long as the induction is kept rigidly within

the same causal whole, like may be induced from like—like cause as complementary from like effect, and like effect from like cause, as also like cause from like cause, and like effect from like effect. When the observations are assured, the great difficulty here will lie in the exclusion of every other cause from operating in the given chemical combination.

§ 174. *Polarity as Cause.*—Observation has ascertained that inorganic matter tends, when its particles are allowed free motion, to assume determinate geometric forms. Here, then, is found a force operating in a definite way, and characterizing itself as a specific kind of cause. In this causal whole induction has its legitimate opportunity. The observed cubical crystallization of one portion of common salt in solution in water is the ground of an induction of like effect in every other similar portion. It is ground also for concluding that other substances in solution will, in becoming solid, assume geometrical forms. The grand difficulty here is that of determining in what precisely the cause of the crystallization lies, and whether the cause be single or otherwise. The more precisely this is ascertained, the more exempt from contingency will the induction be.

The phenomena of electricity and magnetism also evince the operation of a natural force characterized as polar. These polar phenomena, regarded as effects, form with that force to which

they are attributable a causal whole in which inductions may move either from force to phenomenon or the reverse, or from one polar force to another, or from one effect to another. On the assumption, thus, that all these polar phenomena belong to the same causal whole, that is, are attributable to the same force characterized as polar, whatever results are found to attend the operation of this force in electric phenomena we may induce to like results in proper magnetic phenomena. In truth, inasmuch as throughout universal nature this polar force is recognized everywhere, working out results in contrasts or oppositions, whether appearing in new special relations or in attributes whether of quality or of action, so induction may construct polar causal wholes from the more comprehensive down to the lowest specific whole, in which it can legitimately move, and, all errors from other sources being excluded, may attain sure results of truth.

§ 175. *Heat as Cause.*—Observation detects a form of physical energy which is identified by a peculiar sensation when working in the matter of our bodies and which we call *heat.* By induction with the help of a disjunctive exclusion—by excluding other causes in the case—we infer from the sensation of heat in ourselves when in certain relations to the sun, to like effects in other like bodies in the same relations. If our knowledge, however gained, assures us of the disjunctive proposition that this causal force either re-

sides in certain portions of matter or is simply transmissible from one particle of matter to another, then by the exclusion of the former of these disjunct members of the proposition, that is, by proving that the former member is not true, we establish the other member, that is, that the force is transmissible—in other words, we establish the undulatory or dynamical theory of heat. This truth having been attained by inducing cause from effect, we may infer from the undulatory motion in heat a source or beginning of this motion—may induce to its cause. So again by induction we infer from the expansion produced in one body, as a piece of iron by its being subjected to heat, that every other piece of iron will expand on being heated. The induction, it might be supposed, would regularly extend to all material bodies. But it is found that the law does not hold in all cases; water seems to contract through a certain stage of the heating process; caoutchouc also contracts; certain metallic alloys contract. The induction seems thus not to be absolute, does not give necessary or universal truth. The explanation is that the condition of this matter in these exceptional cases is different from that in the other cases, so that the heat-force cannot produce in them the same result. In truth, in the operation of every cause, three different factors enter into the full explanation: first, the force itself having its own intrinsic qualities; secondly, the object on which the force

acts having its peculiar properties; and thirdly, surrounding substances or forces (§ 181). It is this third factor—the environment—which occasions the chief difficulty. It is liable to be overlooked; it affects the operating force, for all the forces of nature are in organic relationship to one another; it equally affects the material body on which the force acts; it affects the operation itself. When due allowance is made for all these forces which may come in to modify the action of the particular force in question, the result may be absolutely true. The inductive principle holds good everywhere. Any contingency in the case must be attributed to the determination of the *datum*. More commonly it will be found to be due to a possibly inexact determination of the causal whole.

§ 176. *Light as Cause.*—Light, like heat, is identified by a sensation as its proper effect. Light and vision constitute the two parts of a causal whole. From vision as effect, if at least certain affections of the visual organism from disease or external irritants, as muscular violence, electric excitements and the like, be excluded, we induce as cause a force acting. By disjunctive exclusion, either the corpuscular or the undulatory theory of light is to be established. The determination of the true theory will facilitate the inductive movement, as narrowing the field of supervision. If, now, for a particular example of induction in this causal sphere, we note the dif-

ferences in time in two apparent eclipses of a satellite of Jupiter, one when the planet is nearest and one when it is farthest from the earth, we shall have the *datum* for a legitimate induction. The velocity of light from Jupiter induces to a like effect, a like velocity in light from any other luminous body. So observations of the angles of reflection and of refraction in single instances legitimate inductions to every other case of similar reflection and refraction. The so-called law of the intensity of light—the law of the inverse square—ascertained in one case, is induced to every other like case.

If we assume the undulatory theory, we have a causal whole determined in which a more general induction may proceed:—as, pitch in sound being dependent on the number of aërial waves striking upon the ear in a second, in other words, on the velocity of the wave-motion, we induce that color in vision in like manner is dependent on the velocity of the wave-motion. When such inductions are confirmed by observation, the results receive a corresponding assurance of their being true, the contingency ever attending the observations or primitive *data* being so far removed. So the law of intensity, that it is inversely as the square of the distance from the source of the wave-motion, when once ascertained in the case of either sound or light or heat, may legitimately be induced to either of the other two classes of phenomena. The same

observation applies to another law—that the intensity varies inversely with the square of the amplitude of the wave-motion in these different classes of phenomena. In truth, the phenomena of sound, light, and heat, so far as depending on the wave-motion of different media, may all be embraced in single causal wholes in which inductions from like part to like complementary part, are legitimate, subject only to the contingencies attending the *data* from which the reasoning starts.

§ 177. *Vitality as Cause.*—The energy which we recognize in nature appears to us to be in some cases characterized as vital. It manifests itself in all living things. We have thus a vast causal whole in which force as vital is the defining measure or limit. Starting from the single observation that the plant comes from the seed, or the bird from the egg, that is, that an individual life springs from an antecedent life, we have a causal whole constituted of the two parts of a force causing and a result caused, by virtue of which both parts are denominated as vital. Excluding by a sufficient number of observations all supposable contingency through the intrusion of foreign force, we arrive assuredly at the conclusion that every newly observed life is from antecedent life and passes into other life ;—*omnis cellula e cellula*, or in the exacter phrase of recent biological science, every blastomere a result from

blastomere as cause; every blastomere into blastomere.

It will be noticed here that as elsewhere the inductive whole is determined by the force appearing in the matter, not by the matter itself. The same amount of matter although similarly related in its parts intrinsically and extrinsically, may not be living. So by a process of logical exclusion we conclude and familiarly recognize the fact that there may be a corpse—body without life. The comprehensive causal whole then here is determined or constituted by energy so far as living. From its recognized presence we induce the result—an effected life in which the causal force still residing cannot die, but ever passes on into new life. In the revealed movement of thought, from effected life, we equally induce a causing life.

In a lower inductive movement in one of those two constitutive parts of a vital whole, induction has a range as wide as living being itself. Thus in the cause, by observing that one life works a life like itself, we attain by induction combined with logical composition the comprehensive law of *kind* or *heredity*;—like produces like; that is, life, as producing or acting as causal energy, if unrestricted and unmodified by extraneous force, ever produces life like itself.

In like manner, in all the subordinate forms of effect resulting from the operation of vital energy as cause, induction is practicable and valid.

Living energy, thus, as causing circulation constitutes a smaller causal whole in which, from observed circulation in plants, we induce to the like in the circulation as it takes place in animal body. Or, still more subordinately, capillary attraction being observed to be insufficient to effect circulation in the vegetable, it is induced to be insufficient in the animal; or positively, vital energy being necessary to circulation in the one kingdom of living being and its true cause, it is equally necessary in the other. So we induce from osmose in the one with all the details of the process, so far as determined unmodifiedly by the vital energy, to osmose in the other. Just so, observed facts in the nerves, in any particular nerve, as the auditory nerve or the visual nerve, in a sensory nerve or a motor nerve, in a dog or in a rabbit, lead by legitimate induction to like facts in the nerves of the human body. Dr. Ferrier thus from experiments on the cerebellum of the rabbit obtains solid grounds for the conclusion that that organ in man has for its function the regulation of vision by harmonizing the diverse organic parts in their action and co-ordinating all with the motor-nerves affected.

§ 178. *Instinct as Cause.*—The energy in the universe appears often with certain characteristics which enable us to identify it as a peculiar form of force and accordingly to designate it by a name—instinct. The tendency in the particles of inorganic matter when free to arrange them-

selves in straight lines and geometrical figures—the crystalline force—as also the polar force giving rise to opposite properties in opposite spacial relations, perhaps also certain reactionary tendencies observed in the natural world, evince the presence of forces which are entirely distinguishable from the mechanical and molecular forces centering in matter, and so far share in some peculiarities of instinctive force. In living being everywhere—in plants, in animals, in man—we discern instinct working conjointly with vital force and with physical force, yet distinct from both, independent of them and often superior to them. We have thus a causal instinctive whole, within the range of which induction has legitimate play. From the discerned presence of the result, we induce the producing energy; that is, from the effect we induce the cause as complementary part of the causal whole; and also from the discerned force we induce the resulting effect. From instinctive force acting in certain conditions we induce instinctive energy in other like conditions; from its producing certain results in one case we induce like results in other cases. In like manner, observing certain results that could not be reached by the energies proper to the being itself, results bearing the marks of intelligence, forecast, design, skill, that do not belong to the proper nature of the being, we induce like results in other beings. Observing, thus, geometrical skill n the construction of its cell by one

group of bees, we induce like skill in other groups of the same species; from nest-building skill in one pair of birds, we infer like skill in another.

The broader and more comprehensive inductions, here as elsewhere, are comparatively exempt from serious liabilities to error because the causal whole is more exactly definable, and there is consequently less room for the intrusion of other forces. As we descend into the lower and more subordinate parts of the comprehensive instinctive whole, modifying causes come in and the precise operation of each with the measure of its modifying power in the case is more difficult to determine. The closer the affinity the more sure is the induction so far as the more specific attributes are concerned. We may more safely induce from one bird to another in the same variety or species than from bird to quadruped or to plant. We have thus an exemplification of the general principle that the narrower the field the more secure from error is the induction.

§ 179. *Person as Cause.*—We recognize an energy which we designate as personal, bearing the marks of conjoined intelligence, feeling, and freedom. It constitutes a causal whole in which are presented in manifold directions manifold opportunities for legitimate induction. The causal whole here is determined in a way exactly analogous to that by which other causal wholes are attained. An attribute is observed, here personal or rational, in some real object concerned as re-

sult of some action, in other words, as effect. This observed attribute marks out a whole of objects alike possessing this attribute. We have thus a causal whole, embracing first and primitively one object, which in one relation as resolved by thought is recognized as cause and in another as effect, and consequently as presenting the two parts constitutive of a causal whole. As other objects are observed to possess this attribute, a class-whole is formed exactly defined by this causal agency, here personal or rational. These objects are like complementary parts between which induction is legitimate.

Personal or rational energy is observed to be characterized as essentially aiming or end-seeking; all rational action seeks an end. But end is specifically modified indefinitely; and accordingly, rational acts, conceived as causes, that is, as end-seeking, or as effects, that is, as ends attained, are determinable by the particular end thus sought or attained. The more specific wholes in this class are to be determined by the more specific end. The more exactly the causal whole, in which the induction is carried on, is defined by this end, the freer and surer is the movement. Within the same scope or range of end or aim, induction is comparatively clear and easy, whether applied to a rational cause or agent or to a rational effect or act. We induce thus pretty safely that if one man pursue a certain end in a certain way, another man in like conditions will pursue

it in a like way. Station, learning, wealth attained by self-sacrificing endeavor in one case, we induce, may be so attained in another case. Here, however, in a peculiar degree modifying forces come in complicating the movement of thought and so requiring unusual care in eliminating sources of error.

SUMMARY OF PRINCIPLES REGULATING INDUCTION.

§ 180. RULE I. *Distinguish the specific object or end of the induction, whether of verification or of discovery.*

In the latter case—that of discovery—if the result be assumed conjecturally, the procedure will be the same as in verification, for which in common the following rules may be given. Some peculiarities in the case of discovery will be noticed afterwards.

RULE II. *Recognize and verify the datum.* This *datum* will be the fact or truth on which the induction rests and from which it starts. The fact or truth may be attained by observation or by proof. It should be carefully tested at the outset as to its truth and accurate representation in language throughout the induction.

RULE III. If the argument be complex, *recognize the inductive movement and separate it from all that does not belong to it.*

RULE IV. *Reduce the inductive part to its simple and regular form.* The *sorites* should be

resolved into its component parts and the enthymeme and epichirema put in full regular form. If the *datum* be an enthymeme in which the sumption is suppressed, the middle or mediating term and the minor term must be given and will form the subsumption; and the conclusion will contain the third or major term. The sumption can then be formed of the middle and major terms. If the subsumption be suppressed, the terms of which it can be constituted will be found, the minor and major in the conclusion and the middle in the other part of the enthymeme—the sumption. Verify the conclusion as to its quality, which must be affirmative or negative according to the sumption.

RULE V. *Reduce the whole in which the induction moves to its lowest or narrowest form.* This rule is of great importance in facilitating the thought and also in exposing errors which in a broader field might escape observation. For securing these advantages a familiarity with the classifications of inductive wholes is desirable. As has been before remarked, it is not necessary to the validity of the induction that absolutely the best or most perfect classification which science can obtain should be followed. The classification only subserves the purpose of facilitating the thought and keeping it from errors that might otherwise be overlooked. The following table of classes may be found useful to this end:

TABLE OF WHOLES.

§ 181. I. WHOLES OF THOUGHT:—A. *General Logical Wholes*, comprising—1. Class or subject wholes, as genus, species, individual, with their subordinations; 2. Comprehensive or attribute wholes, comprising (1) categories of pure thought so called, or general classes of the attributes of thought in itself, which from their very nature must be superordinate to all other attributes (§ 66); (2) categories or generic attributes of the matter of thought; (3) categories of matter thought, cause and substance (§§ 191, 194).

B. *Special Inductive Wholes*, comprising—1. Substantial wholes, with the four subdivisions according to direction of movement in the fourfold relationship of substance to quality; quality to substance; substance to substance; and quality to quality; 2. Causal wholes, comprising those formed by movement of thought from cause to effect; from effect to cause; from cause to cause; and from effect to effect.

II. REAL WHOLES:—1. *Material*, comprising organic and inorganic; 2. *Mental* or spiritual, with subdivisions in both branches as furnished by science.

More subordinate or lower wholes may be attained for the induction by cross divisions, or partitions.

RELATIVE WHOLES.

To the foregoing enumeration, which embraces only wholes constituted of intrinsic properties, there should be added a class of wholes which are constituted of merely extrinsic or relative attributes. They may be either logical or real—wholes in mere thought or wholes in real existences. It is evident that any two correlatives such as greater and less, farther and nearer, former and latter, parent and child, governing and subject, etc., constitute a dual whole, in which induction may proceed as in the other cases of wholes. It may be observed here that induction to the different, as expounded on subsequent pages, is peculiarly available here. Here belongs a class of inductions of great importance to science, those moving in proper conditional wholes, that is, between the whole and the part, as themselves constituting a whole allowing legitimate induction. Thus a naturalist infers the existence of a limb from the known existence of the body, or, conversely, the existence of the body from a limb; the existence of the animal from a discovered bone.

Body and member, whole and part, are complementaries of one logical whole. So the conception of position or place, of *here and there*, or of spacial extension, which arises from simple apprehension or perception of what is truly pre-

sented or given to thought and so external to it, is a conception of a part of a containing whole—the whole of space. From the part—extension—which is limited, as is implied in the very idea of part, we induce legitimately to the whole—space—which thought merely as a whole is thought as unlimited. In the same way, from observed duration, as a part of time and consequently limited, we induce to the conditional whole of time, which so far as viewed as a whole is unlimited, part and whole differing from each other characteristically in this respect, that the one is limited, the other unlimited. Such a whole may, it is true, itself be viewed in another movement of thought, as a part and so limited; but as complementaries of each other, the part is limited, the whole is unlimited. The reasoning thus is valid: If there is extension perceived, it must be limited, for all objects of perception or of sensuous apprehension must be limited; and as limited it is a part and must have its whole, which so far as a whole must be unlimited, as its complementary. From observed extension we attain thus by legitimate induction, unlimited space.

Here belongs also another large class of inductions applicable to the relations between the parts themselves of composite bodies. Thus the phenomena of allotropy and isomerism in chemical combinations are explained by the diverse effects resulting from the divers modes in which

the same elements combine in respect of order, distance, and the like.

Perhaps the most frequent and most important uses of induction will be found to lie in the sphere of causation; certainly it is here that the peculiar, if not the greatest, difficulties will be encountered in scientific discovery. The chief of these difficulties will be met in determining precisely the causal whole, in other words, the exact range and intensity of the force operating in the case. We have already named (§ 175) the three entirely distinct factors to be distinguished in determining this: first, the essential nature of the causal force itself; secondly, the nature of the object on which this force acts, since the results may depend as truly on that as on the cause, for heat-force does not affect gunpowder and ice alike; and thirdly, the environing forces and objects which are present in every phenomenon and ever tend to enter into and combine with any operating force and to modify the result of its proper action. The nature of the force and the character of the object in its relation to this force it may be easy, it is in fact generally easy, to determine; but both exist in relation to other environing forces and objects. We know what heat-force is; we know what gunpowder is in relation to heat; but if no oxygen be admissible, no explosion results from the contact of fire with the powder. If we know even beyond a doubt the nature of each of two forces, so as to be able

to predict how each will behave if operating by itself, we may not be able yet to forecast the result in case the two forces combine. The most harmless salts are obtained from the combination of elements each by itself most noxious. Everything may depend on the way in which the different forces meet, whether chemically, mechanically, or simply in juxtaposition. The elimination of those forces which are extrinsic to that composing the causal whole in a given induction is often extremely difficult. It is sometimes even impossible, at least at the present stage of science. In fact it may be assumed that the resolutions and combinations of forces in the universe with the determination of the precise working of each, both singly and in combination, will ever open an indefinite field of inquiry to the finite mind. Yet science may conquer field after field and win large empires of true and useful knowledge; and its chief instrument must be induction.

INDUCTIVE DISCOVERY.

§ 182. In inductive discovery, the unknown result may often be attained by careful inspection of the middle term in its relation to the major, that is, in its relation as a part to a certain whole which will suggest the minor or complementary part sought, and then the suggested result may be verified by actual investigation. Sup-

pose, for illustration, the question to be raised: Is there another planet outside of the orbit of Uranus? Inductive thought will turn at once to the narrowest causal whole that it can suppose in the case, say, the disturbance of Uranus which is the effect-part in this whole; it will first seek whether Uranus be in fact disturbed. If so, it will then inquire from what source; and by excluding other sources or possibly by better directed observation of the heavens it may attain the desired fact. As rational curiosity must ever be prompted by some particular fact of experience or observation, so it naturally pushes on its way to discovery in the line of this principle of particularity or relationship of parts, expecting to find the like from the like part.

Or suppose again that it be inquired: What is the function of the cerebellum in man? Direct inspection being for obvious reasons impracticable here, the inductive process is naturally resorted to through the inspection of this organ in some animal, say the rabbit, which is a complementary of man as part of the whole class, *mammal*. Experiment evolves the fact that the irritation of the cerebellum of the rabbit by electric excitants puts in motion the motor nerves of the eye. Starting from this observed fact we attain an induction which may be presented in regular form thus: The cerebellum of the rabbit regulates the motions of the visual organ, P is W; man is complementary of rabbit in the class-

whole of mammals, C is P; therefore, the cerebellum in man regulates the movements of the eyes, C is W. There is here, strictly speaking, a twofold induction, as the subsumption involves first an induction from man to rabbit as complementaries and then from the organ in man to a like organ in the rabbit. This last part of the movement of thought is in the causal whole of nerve-force narrowed to that force as working through a particular organ, the cerebellum nerve-force. The awakened curiosity here as before obeys the proper inductive instinct: it finds some complementary part—here the cerebellum of a rabbit—and directing its observation upon that discovers what it may by induction refer to man.

Inductive discovery has, it is evident on a glance, a wide range in scientific study. Modern science is inductive to an extent of which it is itself perhaps but little aware, particularly when it goes beyond the verification of conjectural truth and pries into the dark unknown. An attribute is given and the search then starts to find its necessary complementary, that is, the substance which it in part constitutes. An effect, a change, is observed, and the inductive instinct puts upon the search for the complementary cause. An object, as a fossil bone, is discovered; inductive thought alights upon the complementary organ; and the archaic bird or beast is restored in size, in form, with the food and general condi-

tions of soil, climate, plants, and correlated animal life that its subsistence involves; with indeed all their antecedents and consequents. Once started the pursuit is limitless, even as the universe is limitless. The complementary of a quality or an effect, of an organ or a function, of an individual or particular, is ever a legitimate object of search, till the entire whole to which they respectively belong is reconstituted from all the parts. This complementary may be but an inconsiderable thing in itself; but it has its complementaries of other ranks and orders, and may bring into view, through the necessary correlations of parts, fields of objects stretching away to the uttermost boundaries of human knowledge. And if the observation which is the *datum* of the inductive movement be true and the induction itself legitimate, the results must be accepted as valid. If, moreover, the curiosity to discover be guided aright along the fixed laws of knowledge by the simple regulative principles of inductive method, the pursuit may be in light and assurance and so in comparative ease as well as satisfactory in wealth of result.

Mr. Mill in his Logic has presented five "Canons of Induction" which are named respectively: 1. The Method of Agreement; 2. The Method of Difference; 3. The Joint Method of Agreement and Difference; 4. The Method of Residues; and 5. The Method of Concomitant Variations. These canons obviously do not bear

directly on the proper inductive movement, they are however practicable helps in the attainment of the *data* for induction by observation and experiment. The name—'Canons of Induction'—might mislead as to their true character.

§ 183. INDUCTION FROM FORCE OR ENERGY TO A BEING.

When on actual observation there is given to thought a force or energy, the question naturally arises, Can rational curiosity rest with merely asserting the reality of this force, or with its causal dual of beginning and ending force—with a real cause and effect? Can thought, in other words, rest in the recognition of mere attribute? The real gives itself to thought ever and necessarily through an attribute; but it gives itself in its concrete wholeness; it is not mere abstract attribute. It is a grand mistake to suppose for instance that power can be given to us apart from a being to which the power belongs. But the real is ever a single and a simple; it is thought only that, by its analytic function, makes of it a dual and a composite; and it is equally the function of thought and the necessity of thought if left to its own prompting that, in its synthetic power, it again unites the double it has made and affirms the dual subject and attribute to be but one and the same, constituting out of these as complementary parts of thought one actual

whole. So here logical induction finds its proper field and may work unchallenged its proper work. Universal thought and universal language among men have recognized force or energy as attribute only; never, save in the airy flights of rhetoric or poetry, exalted it to the rank of concrete being; never but in the flounderings of a blind logic or a blinder metaphysic confounded it with substance. Or to take the other view; force, energy, power is ever recognized as the essential element in every operative cause, of which we cannot conceive but as residing in some active nature, that is, in some form of concrete-being. Physical cause is by universal consent of learned, scientific, and illiterate laymen traced to some material body from which its operative force proceeds; and cause from will, intelligence, or imagination is as universally derived from a real personal being, for even a sheer materialistic philosophy will not reject the appellative of personality as applied to mental phenomena even when denying everything back of organism. Energy, force, or attribute, is the complementary of substance in the thought-whole of being. Given accordingly an instance of real force, of energy, and logical induction infers its thought-complementary, that is, the substance to which the attribute belongs. The real being is thus affirmed under the very necessities of thought; for real attribute and substance are but the dual into which analytic thought originally resolved the single concrete

being, and synthetic thought now reunites this dual into the one concrete being of which they are but the thought parts.

The being, to which inductive thought thus necessarily leads when an instance of force as attribute is given to it, must of course be characterized conformably to the character of the energy observed. If this energy be observed to be finite, only a being correspondingly finite can be inferred. If the energy be recognized as supreme, as universal, as without discernible limits, the being induced must be so characterized as so far supreme, universal, boundless. If there be a sovereign, comprehensive energy observable in nature, then in nature is there a being correspondingly sovereign and all-comprehensive—a supreme, an almighty One.

INDUCTION TO THE DIFFERENT.

§ 184. Hitherto for the sake of simplicity of exposition we have viewed the inductive movement only in the line of sameness—from like to like. But there is evidently in the relationship of part to part another movement of thought in which we conclude from one of two complementary parts not to a like but to an unlike. For it is very obvious that while any one part must be the same in some respects with any other part, it cannot be the same in some other respects; for if it were, the two would be abso-

lutely and totally indistinguishable, and consequently the same; there would be not two, but only one. In fact, under the general truth that in thought we may legitimately affirm or deny, identify or difference with equal validity, we are justified in presuming just this pair of movements in induction—to the same and also to the different. It may be serviceable to illustrate the nature of this species of induction in different leading departments of thought.

In the primitive form of thought—a norm of pure thought as *A is B*—we have two terms A, B, identified, indeed, so far as predicate can be with its subject, we may say, in loose discourse, absolutely identified; but yet in so far as they are respectively subject and predicate, they are different: A is subject and B is not; B is predicate and A is not. We induce accordingly that if of the two terms A is subject, then B is not, but is different; or if B is predicate, A is not. If we inquire now what is the particular respect in which subject and predicate differ, in order to determine where in an induction we are to identify and where to difference, we find it in the essence of the proposition itself which makes them respectively subject and predicate, and this essence is found in its assertive quality;—asserting the predicate of the subject, not the reverse. In other words, the ground of the difference is found in that which constitutes the two into the whole—the proposition, the very essence of which

is that it creates two opposite different relations, —*thought* and *thought of*.

In a mathematical whole, say a geometrical whole of a square, if we take two like parts, as those which arise from bisecting it by a diagonal, we may induce the same from one to the other, as in figure—triangular, isosceles and right-angled, and also in dimension—half of the bisected square, so that having found the contents of one we know the contents of the other assuredly without more calculation or measurement. But the two parts are also different; and the difference here lies in the nature of the whole of which they are parts. It is a spacial whole and the parts are different in respect to the spaces they respectively fill—one is to the right, the other to the left; one positive, the other negative—in respect to the bisecting line which produces the parts. We induce to the different when we infer from one being to the right, or positive, that the other is to the left, or negative. We look here as before to that in the whole which originates the parts to find how to discriminate what is same or like from what is different or unlike in the parts.

Class-wholes are constituted by thought itself. Being formed on the basis of a common attribute their logical parts are alike so far as that basic attribute appears in them; in all those respects, in which one part is distinguished from another part, the induction must be to the different. All subdivisions of the vertebrate kingdom, thus, are

alike in having a bony column made up of distinct portions or segments and a nervous system. Any individual of any variety is like every other in this respect; but in respect to any specific distinctive character or property there can be no induction in this class-whole, except to a different. The difference may be under the same order and so relatively to each other either numerical, as if A is first B is second; or spacial, if A is here B is there; or chronological, if A is the antecedent B is the subsequent; or causal, if A is the parent B is the offspring.

In logical comprehensive wholes, the qualities that constitute them in any particular case are alike so far as sharing in the comprehensive character of the whole which they constitute, but are of course unlike in their respective characteristics. If, thus, *intelligent*, *feeling*, *free*, be the qualities which together constitute the whole *rational*, and if *rational* be *active* and *end-seeking*, then those qualities must be alike in these respects; but if *intelligent* be distinctively *discursive* and *analytic*, *feeling* and *free* must be different; they are not *discursive* and they are not *analytic*.

In causal wholes we have with the identities or likenesses also such differences as these: the cause, so far as cause, is different from the effect; one part of the effect from the same cause is different in some respects from each other part.

From the rapid survey which we have taken of

the different kinds of inductive wholes, it appears that induction to the different ever accompanies induction to the same. In other words, where one is practicable the very nature of thought as alternative allows the other. But we cannot properly induce to anything more than to a pure logical different, that is, to a mere negative. This negative may, however, by other processes of thought and in combination with other truths, be helpful to knowledge through the different grades of logical oppositions, as pure, contrary, sub-contrary, and so on.

The important truth elicited from this survey is that in inducing simply to the different, we gain a logical negative term, whereas in the other form of induction we gain a positive.

DISJUNCTIVE INDUCTION.

§ 185. It is perhaps in proper disjunctive wholes that a right exposition of this form of the inductive movement of thought—induction to the different—has greatest significance for science and philosophy. We have the highest and purest form of a disjunctive whole in the disjunctive judgment of the form: *A is B or it is not B.* Here is a whole having for its parts the two disjunct members, *A is B* and *A is not B.* They are complementary parts; so that from the one we may always induce to the like in the other; as if *A is B* is cogitable, *A is not B* is cog-

itable; for they alike participate in the cogitable character of the whole—*A is B or it is not B.* We attain thus that fundamental and wide-reaching principle of thought that contradictions are equally and alike thinkable. If we may think that *A is B*, we may think *A is not B.* This is a principle, it may be remarked, which effectually undermines and explodes the fallacy not infrequently vitiating recent speculation, that what is not true cannot be thought or conceived,—a fallacy indeed simple enough, and not rare in experience, for the wildest chimeras have been conceived, and have been dangerously insidious.

But we may induce here to the different:—It seems simple enough to say that *A is B* is different from *A is not B.* And the difference is precisely the difference between the affirmative and the negative; so that if *A is B* is affirmed then we induce at once that the disjunct member *A is not B* must be denied. We have thus the principle of the familiar disjunctive reasoning. It is a principle so extremely simple and so familiar that while it is freely employed in all scientific investigation, the failure to recognize its precise nature and value as guide and authority often occasions stumbling and error. Notably it has been widely confounded with another form of disjunctive whole and its sovereignty in ruling thought has been usurped and fallaciously exercised by a mere subordinate.

This lower form of a disjunctive whole may be

thus formalized: *A is either B or non-B;* or *Space is either finite or infinite.* The former case was one of pure contradictory opposition, in which there appears an alternative in the thought itself represented in the copula, so that it is of the necessity of thought from its very nature that we must in thinking either affirm or deny, and if we affirm we cannot deny the same that is, we cannot affirm the contradictory, but if we think it at all, must deny it. But in this case the disjunction is not of pure contradictory opposition; it does not lie in the thought but only in the matter—the terms—of the judgment. We may legitimately induce from one of its parts *A is B*, *Space is finite*, to the like as the complementary part; as, if B is a thinkable attribute of A, non-B is also a thinkable attribute of A; if finite may be conceived of space, infinite must be conceivable of it. But it may be that neither of two contrary attributes, neither *B* nor *non-B*, can be ascribed to a subject. For example: *The copula of a logical judgment is neither finite nor infinite; a stone is neither vertebrate nor invertebrate.* Here we detect a fallacy amazingly frequent in recent philosophical and scientific discussion. It is forgotten that thought is utterly incompetent to create *àpriori* any attributes whatever except those which it discerns—intuits in its own nature and forms—any attributes except the so-called categories of pure thought; and even these must be observed before they themselves can be thought

and can be observed only in an actual thought. Only what is given to it can it exercise itself upon; it cannot create its own *datum*. The absurdity of supposing it seems so gross that it would be thought none would attempt it. Yet what else are all those attempts to construct *àpriori* theories of matter, of space, of time, of God? Discursive thought, we repeat, is utterly incompetent to form a properly primitive *datum*. The first thought must have an object given it upon which to exert itself. It is to be remembered, however, that a single thought, any single thought, must contain the entire essence of thought, and therefore in every instance of thought all that essence may be observed and then may become a proper *datum* to thought in all its attributes; and thus the laws and categories of pure thought may be attained by simple inspection of what is given, and then of course if any object is to be thought, it must be thought in subjection to these laws and attributes, and therefore must possess all these attributes, as otherwise they could not be thought at all. But with the recognition of this peculiarity in the case of the categories of pure thought what can be conceived of more grossly absurd, more entirely contradictory to the entire nature of true thought, than to assume some utterly unknown object—say X—some object which has never revealed itself or been revealed by any attribute to the mind's apprehensive nature, and assert of

it that it is this or that—that it has this attribute or has not that? In repetition, and with emphatic earnestness, it must be held absolutely, always and everywhere, that thought can furnish no attribute whatever, except its own essential attributes attained from inspection of its own operations, in the case of any object whatever exterior to itself. Whether X is finite or infinite, vertebrate or invertebrate—one or the other—thought is utterly incompetent to determine till X reveals itself in some attribute. All that speculation, philosophical or scientific, that undertakes to construct theories of anything before that thing has been given or presented to the mind's apprehension in some attribute which belongs to it, except only, as stated, the essential attributes of thought itself, is baseless and worthless, or worse, is essentially fallacious, however blind to its own fallacies.

To recur to the form of disjunctive induction under consideration, it is evident that it differs widely from the first case of pure contradictory opposition, in which it is of the necessities of thought that if the former of the two members be denied or sublated, the latter must be affirmed or posited, for the opposition is in the thought —the copula. But in this case, the opposition lies in the matter of the thought, the terms, and to affirm the second member on the denial of the first involves an addition to the matter of the thought which is not allowable. This is clear

from the examples, *A is B or non-B; space is finite or infinite; a stone is vertebrate or invertebrate;* if we deny or sublate the former disjunct member in these several cases we do not necessarily affirm or posit the latter of the two members; we cannot in other words induce that *A is non-B*, or that *space is infinite*, or that *a stone is invertebrate*. The different to which we can legitimately induce is simply the negative, the contradictory, in other words the different in pure thought. If we go beyond this we fallaciously affirm an attribute to exist which has not been given to thought. There is something positive gained in the induction, it is true; but this character of positive attaches only to the copula, the assertion; it is fallacious to extend it to the attribute. Otherwise, if we put in the first of the examples given—*A is B or non-B*, *A = tree* and *B = xyzs*, then we might reason: *A tree is either xyzs or inxyzs; but it is not xyzs; therefore it is inxyzs*. We have in this way proved to us, that *inxyzs* is a positive attribute and must therefore belong to something as real as itself, although this something is other than a tree. This fallacy is notably exemplified in the argument for the being and attributes of God; as, *God is finite or infinite;* but he is not *finite; therefore he is infinite*. The conclusion may be true; but its truth is unwarrantably inferred from the premises, unless the subject—God—is understood to be a real being; for the being of God not having been proved in

other ways, or at least not legitimately assumed in the reasoning, there is a surreptitious bringing in of reality in the form of an attribute which although primitively merely negative is construed in the conclusion as a positive, thereby implying the reality of the being to which it belongs. The fallacy is a contravention of the very first principle of thought that its sole function is to identify or difference, and so it is utterly incompetent to add to its *datum*.

It is still to be maintained that this form of inductive thought may be of most serviceable value in the advancement of knowledge, provided only that the fallacy, so common in its actual use, be fully guarded against.

There is a third class of disjunctive wholes which is distinguished from the foregoing by involving only a lower degree of what is called in logic *contrary opposition*. In this case the disjunct members or complementary parts are incompatible attributes, so that if one belong to the subject the other cannot; as *the periphery of the sun is circular or elliptical; A is B or C*. Here induction to the different is obviously legitimate, provided only that the presence of the one attribute in the necessities of thought or by other proof excludes the other. If it be established that the sun's periphery is either circular or elliptical, then the induction is valid; but not otherwise; for it does not necessarily follow that it may not be of some other figure.

We cannot assume without grounds that *A is B or C*, and then on proving that it is not B infer that it must be C; for A may be D. Here too the inductive movement to the different may be most serviceable to science, but it is liable to fallacious uses. Here, as in the second class of disjunctive wholes, the disjunction assumed in the sumption must be verified, so that by the necessity of thought no third thing can be supposed; and, moreover, it is necessary to guard against admitting more into the matter of the conclusion than is introduced in the premises.

THE ULTIMATE GROUND OF REAL INDUCTION.

§ 186. It is popularly held that the ground of induction in the world of reality is 'the uniformity of nature.' But this maxim that nature is uniform is at best only a half-truth, for nature is as truly diversified as it is uniform. We find here no sufficient ground for scientific thought. It is obviously inadequate; for induction is as valid when it concludes to the different as when it concludes to the same. Moreover, the act of thinking belongs to the world of reality. From any such act of thought induction is possible, before the outside world has come into the experience. Not that this is historically the case—that we exercise thought in fact before any experience from without; sensation prob-

ably presents the occasion for the first movement of thought-activity. But in the nature of things induction is possible before any such experienced sensation and therefore independent of it. Mental activity may, for aught we know, be first awakened by an inner spiritual force. Induction cannot rest, consequently, on external nature for its ultimate ground. Still further, if it be asked how we know that nature is uniform, the only possible answer is: we know it by induction. To ground induction consequently on the uniformity of nature is to ground it upon itself. But that the validity of inductive thought, and indeed of all thought, must ultimately rest upon its own nature is little else than a self-evident truth. It cannot depend upon its objects—its *data*. For if nature were to present only phantoms, there might be genuine and valid thought on them. They might be recognized as phantoms, and numberless relationships might be thought among them as impressions on the mind —between themselves, as in their several natures, and also between them and the mind, and other objects of thought. If we ever make the unallowable supposition that nature, that the world of reality, should present to our thought contradictories, even in that case thought, inductive thought, could move on one or another of these contradictory phenomena so far as possible objects of thought, and with results that should be certain.

Thought, indeed, cannot wholly determine the nature of its own *data;* it can take only what is given to it. But it cannot take what is contrary to its own nature; it cannot take what is not thinkable. Nature accordingly is absolutely nothing to thought except so far as it is thinkable; except, in other words, so far as true. But nature everywhere discovers truth; it ever moves in lines of truth; nature is uniform because it is true; because it moves in obedience to truth, that is, in obedience to the laws of thought. The uniformity of nature itself has its grounds consequently in thought; it is uniform, as we have just said, because it is true; because it is thinkable or conformable to the principles of thought. Not on what is observed, not on what is presented to thought, accordingly does the truth of thought and the validity of induction ultimately rest; but on the nature and conditions of thought itself. Its results, then, so far as legitimately attained, have no precarious dependent existence. Thought, knowledge is its own master; its rule is absolute; and nothing without can shake its authority. Human thought, however, it is ever to be remembered, is finite, is imperfect; it often stumbles, often errs. But this can never be except as the result of offense against its own laws. Legitimate thought is only another name for truth; and truth is never at war with itself; and asks of no foreign power a sanction to its unappealable decisions.

PART III.

THE RELATIONS OF THOUGHT.

§ 187. We have considered hitherto human thought in its intrinsic attributes—its essential nature as seen in its laws and generic forms or processes. This view has been generally accepted as comprehending the entire field proper to logic as the science of thought, even when the treatment has not been limited to a special movement of thought as Grecian logic limited it to the syllogism. But no subject of knowledge can be regarded as fully surveyed and explored that has not received at least a bounding or definitive consideration of its outer relations.

Thought sustains one vital relationship of this extrinsic class, which is as comprehensive as it is necessary to the very being of thought; which, further, determines specifically and shapes all the divers movements of thought. This relationship is that which it bears to *the true*. The true is the one all-surviving correlative to thought—as the sky to earth. They are to each other necessary correlates. Each involves the other. For thought is the product of an activity that cannot act save upon its object.

§ 188. This term—*the true*—is to be taken in

its largest sense, as embracing all that can be object to the human intelligence, and so including the imperfectly as well as the perfectly true and even what we call the false. For there is nothing positively and absolutely false; in some respect, in some view, every judgment, every proposition contains an element of truth. The adjective form, *false*, expressing an attribute, is not identical with the absolutely *not true*. Of every form of words, as well as of every conceivable object existing in nature, it can be truly said under the necessities of thought, *it is true* or it is *not true;* but it cannot so absolutely be said *it is true or it is false*. Thought deals in perfect accord with its own nature, when it accepts as its object what is loosely denominated *the false;* it is in some respect, but imperfectly, *the true* in its widest scope.

The true is a synonym of *the intelligible* or that which engages or interacts with the intelligence; also of *the cognizable* or *knowable*, as that which engages the cognitive or knowing faculty; and of the *conceivable* or *cogitable* or *thinkable*, as that which engages the thinking faculty; *the intelligence* being but another name for the cognitive and the thinking faculty.

The true as the necessary correlate of thought ever supposes and involves this relation. But it admits divers aspects and is accordingly used in divers modifications. We regard an object, thus, as true in itself if it be attributable and quanti-

tative, that is, if it be composed of congruous parts united in one whole; as we say, 'it is a true orange,' when it has all the essential attributes of an orange, as real, spherical, yellow, pulpy, juicy, etc. When the view is directed on the relation of object to thought, we speak of *the true* as that which is conformed to its object, as when we say, 'it is a true picture.' When the view turns rather on the result of their interaction, we speak of that as *the true;* as when we say, "his conclusion is true." We have, thus, these three modifications of the true—*in* object, *to* object, and *from* object, as well as the general import expressed in the relation—*for thought*—in the sense that the true is wholly and only for thought (§ 27).

§ 189. The sphere of the true as the exact correlative of thought is accordingly coextensive with that of thought. The true is in extent exactly what is thinkable. We do not say that they are conterminous; for this phraseology implies or suggests a limit; and to thought, even to the finite thought of man, there is no limit to the true or the thinkable; the true is not bounded or finite, for no human thought can assign a limit. Neither in absolute logical strictness has human thought any authority for asserting that the true is infinite. Certainly as a *datum* to thought it cannot itself bring to thought this character. Nor can thought itself impart it to its object; it can impart no attributes except such as attach

to its own nature. Thought may pronounce: *the true is finite or it is not finite;* and sublating or denying the former member of the disjunction may posit or affirm the consequent: *it is not finite;* but this is far, as we have had occasion to see (§ 27), from pronouncing it *infinite;* for while the former expression denies an attribute, the latter affirms one which, though negative in form, is not a purely contradictory opposite of the positive form, and implies a corresponding subject—some being, some existence, some concrete. But here none is given or warrantably implied.

§ 190. As the object and exact correlate of thought, the true must admit of the applications to it of the proper essential characters of thought;—it must be that which can be thought under the categories of thought—of identity, quantity, and modality. It must, as it is brought into thought, constitute accordingly a universe, that is, a whole with parts congruously united. THERE IS A UNIVERSE OF TRUTH AS OBJECT TO THOUGHT. It may be more or less extensive and more or less comprehensive relatively to different minds: it is not limited to any. This universe implies that every part may be thought in its relation to every other part as well as to the whole; and consequently in it everywhere there is free room for the great movements of thought, deductively from whole to part and inductively from part to part. There is no fallacy of the circle in this mode of representation; as if we should sim-

ply say, the object of thought is deducible because we deduce. It is rather as if we should say: if we deduce, there must be something which we deduce, and which, when deduced, must bear the characters of a true deduction. Thought must have an object: and that object, in order to be thought and as it is thought, must have the essential characteristics of thought. The true, thus, is a universe of congruous parts and admits all the movements of thought.

§ 191. As such correlate of thought, moreover, the true interacts with thought. It engages thought; calls it forth; induces its action upon it. It is consequently *a real.*

All attribution, all thought, requires as its necessary condition some apprehension—an impression made upon the apprehensive sense from without the particular exertion of thought upon it. Thought must have an object; and thought is utterly incompetent to create its object. This object or *datum* may be from within the mind itself—an intuition as a previous thought, a memory, or an imagination, or a form of the mental activity; or it may be from without the mind—some impression on the external sense—a proper perception. From whichever source originating, from within or from without, the *datum* is an impression produced, and consequently an impression received. There is an interaction between some form of the mental activity itself or of some external object on the one

side and the activity of thought on the other. Whatever thus acts upon the thinking nature is a *real;* is a thing. This is the one significance and sign of reality. THE REAL *is that which impresses the apprehensive sense.* We have thus the category of *reality.* It is the fundamental character of all *data* to thought in their relations to thought itself. As interacting with thought and impressing it, every such *datum* is a real by an absolute necessity, inasmuch as a *datum* is necessary to thought. If thought is real, its correlative object must be real: otherwise there could be no interaction, and hence no *datum.*

It is true that this real, which object is thus extrinsic to thought, interacts with the thinking activity through what is called *the sense.* But the sense is none other than the thinking nature itself in its relation as recipient in this interaction. In the case of what is known as physical or material existence, there intervenes between the thinking nature and the external object what is called the outward physical sense, and only through that does the object reach the mind. In the interaction between a remote object, as a star, and the thought, there may intervene still further an auxiliary instrumentality, as a telescope. Other media may in different cases occur, as a mirror, a vibratory atmosphere or ether, and the like. But directly or indirectly, immediately or mediately, in itself or representatively, the true as the object of thought reaches the thinking

nature through the sense, which, as stated, is that nature viewed as recipient in the interaction.

The questions have been raised: Is this so-called sense trustworthy? How do we know it to be so? May not all that it delivers to us be mere illusion? In fact, may not the whole current of our thought be merely a phantasmic sweep or flux, causeless so far as we know, meaningless, void, a meteoric gleam, more unsubstantial even than that, a show without substance, and utterly destitute of all reality? Such questioning cannot legitimately come from any thinking nature: for it presupposes in the questioner the very reality it questions. And the questioner can urge no valid plea in support of his questionings. Indeed, by his very questioning, he admits their utter groundlessness. The cause of human speculation does not indeed run smoothly; difficulties appear and ever-seeming contradictions. But as well affirm that a man is dead because he suffers an ache at times, which a true and perfect life forbids. Call truth a phantom if you will; but if the phantasm has all the characters and the results to our experience of real truth, you have only changed your nomenclature: the fact, the truth, remains the same. Distrust the sense as you may: you know you are impressed; and that the source of the impression is not always within yourself. The instrumentalities and media through which the outer real reaches you may be imperfect, inadequate;

but they convey something, bring some impressing energy from without to you; that is real, even if it be imperfect or obscure. The true is, if the imperfect is; and more, it is affirmed in the very consciousness of the imperfection, for if we are conscious of the imperfection we must be able to conceive the perfect. The mind in its function of receiving impression from without and of thus contributing its part in the interaction between it and its object, the mind, in other words, as passive and recipient sense, is as trustworthy as in its more positively active side of impressing or producing. It cannot act but as it receives; its very activity involves receptivity; it is active only as it is passive. This is from the necessities of the case; the very nature of interaction involves it. And consciousness notes the one as well as the other. It is as competent to observe the impression received as being not from itself, but from without, as to observe the impression which itself imparts. This is a simple fact of experience to be ascertained by simple inspection. If by any supposable possibility a doubt may be aroused in the case of some rare experience, the doubt must be dissipated in the full bright light of a prolonged experience multiplied in an indefinite number of conscious subjects.

The notion of reality or existence is, according to this exposition, a universal as it is an absolutely necessary attribute involved in every con-

summated thought. It belongs to the *datum*, as the necessary condition to thought. It is not created by thought; for thought creates nothing, and the *datum* is prior to thought. It is recognized as real inasmuch as it interacts with the thinking activity. It is not an innate idea: it is not a notion absolutely antecedent to actual thought,—not an *àpriori* idea in the higher and stricter sense, an antecedent condition to thought. It is in no proper sense an original form of thought. It emerges only on experience—experience in actual thinking. It is discernible in all thinking, just as a physical attribute is discernible in a physical object; it is a matter of observation. Moreover, the very notion and proof of the real implies as a necessary characteristic of it that it is *active*. It is only by its activity that the real reveals itself, that it impresses the mind, interacts with it, and engages the activity of thought.

§ 192. All realities may be classed as inner or as outer. The former are mental or spiritual; the revelations of consciousness; they are intuited. The latter are physical that engage immediately the external sense; they are perceived. In so far as a phenomenon of either class engages the thinking activity, it reveals a reality. The creations of the imagination it is to be observed, however, are in another and lower grade of classification often and purposely opposed to the realities of nature. Still in the highest sense,

an imagination as engaging the activity of thought, is a reality, the imagining is real; it acts on the thinking nature and occasions a reaction in it. There can be no more exact definition of reality than this, that it interacts with other real natures. If thought is real, its *datum* is real, even although it be the impression of an act of the imagination.

§ 193. This correlative of thought, the true, constituted as a universe of organically congruous parts, which interacts with the human intelligence, and so proves its reality as well as its activity, and which has to that mind an extent with no limits, has also a content as limitless to human thought. It is in this content most truly not bounded, not finite, for the activity of human thought, in all the supposable enlargement of its capacities, can never apprehend it as it can never compass it; while yet, as we have abundantly seen, no warrant can be discovered for pronouncing that in philosophical strictness it is infinite. It everywhere interacts with thought and engages its characteristic reaction. Its specializations, its particularizations, and its individualizations are limitless in number and in kind. Thought everywhere finds what to it are parts, only parts, but parts of wholes of ever enlarging spheres, extending outwards from the center to a circumference never to be reached, and parts moreover congruously connected as organic parts. It engages one or another of those parts;

accepts it as it is, as it presents itself, and reacts upon it in its own characteristic modes of action. It recognizes it under the relation of subject and attributes; and identifies or differences according to what is given or presented to it.

§ 194. Out of the meeting of thought and its *datum* emerges a third thing, which must in strictness be regarded as neither pure thought itself nor pure *datum* or object, but fusion as it were of the two—a proper *thought-thing*. It bears the characters of *identity*, *modality*, and *quantity*, by reason of its being a product of the action of thought on its *datum* or object, and also the character of *reality* by reason of its being a *datum* interacting with thought. But every such product has an additional character, imparted to it in the very necessities of thought. As all thought is essentially attribution, and all attribution is necessarily either through intrinsic or extrinsic attributes, every product of thought must be recognized as bearing the character of attributive *relativity*, or that existing between subject and attribute.

This comprehensive character is impliedly involved in the first category named—that of identity. But a subordination of this character, which is of special importance to ready and accurate thought, is that which is founded in the intrinsic relationships of thought. These are of two kinds (§ 167), causal and substantial. If any object of thought, any real object, be actually

thought, such a thought-thing may be viewed more prominently in its thought-characteristics, as those of attributableness, and so it becomes a *substance*—that which has attributes; or, again, it may be viewed prominently in its object-characteristics, as those of reality and activity; in this view it becomes a *cause*. We have thus what may be called the categories of *substance* and *cause*. These are categories which have been generally recognized in some way, for the most part however but incidentally, both by logicians and metaphysicians.

What it is specially needful to notice here is their genesis from thought and the particular characterization they receive from this source. These categories, thus, are the product of thought itself; they do not exist in the *datum;* they do not appear immediately in the essence of thought, nor in its constituents. They are derived from the necessary relationships arising in actual thought upon its *datum*. Cause and effect are accordingly correlatives, as are also substance and quality. The one correlative involves the other by the necessitations of thought. This relation of necessity is thus the creation of thought: it does not exist in nature. "Our senses," says Reid, "testify only what is, and not what must necessarily be." A common but very pernicious fallacy arises from imputing to nature a characteristic which properly pertains only to thought. If in common popular discourse this

discrimination be not strictly indispensable, in the fundamental philosophies of thought and thing it is of vital necessity to true results. The truth is all-important here that substances, causes, do not as such, with the relations involved and necessitated in thought, exist in nature. Things exist; these things we take into our thought and we there invest them with our thought-relationships.

In order to guard against any possible misconstruction of this presentation it will not be amiss to call attention distinctly to the fact that this presentation is purely from the logical point of view. What the constitution of nature may be further than what is necessarily implied in logical science, which requires that it must be such that it can be thought—must be thinkable—it belongs to other departments of science or philosophy to show. It may be within the competency of ontological or metaphysical science, and we may say here we doubt not that it is within its competency, to show that all nature—the universe of real being—is constituted, disposed, and ruled in thought—in perfect thought—in absolutely perfect wisdom, so that there can be in it no real contradictions, no incompatible incongruities; no combination of contrary qualities in objects, no checks or reversals or misdirections in the sequences of nature—that the laws of substance and quality, of cause and effect are consequently imprinted in legible characters on all

things throughout the realms of nature. But these thought-laws are not conveyed to our minds by the apprehensive sense. The sense gives us things, facts, not laws. We are forbidden accordingly to attribute to nature as essential to it, the attributes that belong to our thought alone. The thing itself, impressing our sense, is other than the investiture in which it appears to us as thought-thing. What the ontological study of nature may add to what the nature of human thought necessarily demands in order that the things of nature may be thought at all, it is not for logic to prescribe. Far be it from logic to claim that the thought divine that constitutes nature must be the same as the thought human that interprets nature.

Further, the nature of thought does not predetermine at all what shall be the particular attributes, the properties or relations that actually exist in nature. Logic oversteps its bounds in any attempt to impose on things any characters whatever, except these two: first, that to be thought these things must be constituted in their intrinsic character and in their relations so as to be thinkable; secondly, that they be real and accordingly active so as to interact with the thinking activity and thus be able to present to it its necessary *datum*. If there be contradictories in nature, thought cannot entertain them as actual but only as conceivable; and so it may seek to disprove their alleged existence, or to reconcile

them, or to wait in doubt as called to a task beyond its power. And so of contraries; it cannot accept an object imagined to be constituted of incongruous properties or relationships as real. Such a real can be no accepted *datum* to it, except only as conceivable, and so to be disproved, reconciled, or suffered to lie outside of its own realm of attained truth.

But the fact of thought, a single instance of thought, implies by the very necessities of thought a power exterior to thought to give thought its *datum*—a power real, active, impressing, beyond the control of human thought to determine what its impressions shall be; a power consequently that is not finite in time or place. It is the highest and most sacred duty of human thought accordingly to accept unquestioningly what is thus legitimately and truly given it from without and impressed upon its apprehensive nature. Such is its relation in dependence and in obligations of active duty to truth.

Logical science as the exposition of the exact nature and province of human thought clearly evinces how and what it may with absolute certainty attain from any instance of thought, from any conception of real things which the mind may have come to experience, just what is contained in it belonging to the outward reality. In interpreting this out of such a thought or conception, it must necessarily drop off what in thinking it has taken on and attached to the ob-

ject given it :—it must drop the dualism of subject and attribute and take the single thing presented to thought and resolved into this dual ;—it must drop off also all the mere attributes of thought which have been attached to the object in thinking—all those of identity, of quantity, of modality or necessity, so that the object as real and in itself will no longer be the same as anything else, no longer bear the characters of finiteness and circumscription or of divisibility—of quantitativeness in any respect, or those of necessity or contingency in which thinking has invested them. At least the nature of human thought does not allow us to affix those pure thought characteristics to its *datum*. Whenever it does so and puts its own forms on the real object, it involves itself in contradictions and absurdities. Every instance of thought necessarily implies however, as we have seen, the existence of a real, characterized as not only actually existing, but also as an energy or force, that is as active, impressing itself on the apprehensive thought, with a nature that is peculiarly its own by which it may be known and distinguished from every other object. So much thought must recognize in its object ; and the fact that I think necessarily involves this, that there is an object, external to my thought, having its own peculiar characters by which it is distinguishable from the multitude of other objects that may engage my thought. In a word

the fact of thought gives by necessity a real that is exterior, individual, and peculiar.

§ 195. The real, becoming the true by being thought, and so becoming invested with the proper characteristics of thought, becomes quantitative; it may ever be viewed in the twofold relationships of quantity, those of whole to parts and of parts to parts. This quantitative treatment of the real universe is accordingly under the control of thought. It is imposed upon real nature by thought. Nature is not in itself thus partitioned off as the needs, not to say the caprices, of thought may dictate. Thought as organic part of a free rational nature, when called to interpret its revelations, selects within limits prescribed to its finite capacities from these revelations its own *datum*, accepting or rejecting at its own will what is presented from without and of itself taking up anywhere what it may choose, of the real universe with which it is in interaction. It draws its own lines of division or partition across the unbroken continuity of the real; and takes up just so much as it may please of the stretch of substance or of the stream of causal working. It must obey the unrelenting and all-prevailing principle of its being, forbidding it ever to transcend its *datum*, adding nothing except in the way of shaping and investing it with its own forms and apparel. Every particular portion of the substantial universe and every particular portion of the flowing universe must

retain its full essence in its own proper purity. To mix and confuse its *data* is to it a radical vice. The particular will thus in legitimate thought be ever kept in possession of its own essential properties and be sharply discriminated from every other particular. The sciences of the real will vary accordingly with the variety of the particulars accepted by thought for distinct consideration and will bear each its own special form and character.

Each particular part of the real accepted by thought, each particular object in the universe selected for scientific treatment, must of course possess this one characteristic in common, more or less explicitly, as yet fundamental and essential, that it is to the very core pervaded with energy, power, force, by whatever term the notion may be designated. Every part of the universe apprehended in thought, every object, every event, is in its innermost center active, embodying and expressing energy or power. It could not otherwise be a *datum* to thought; could not interact with the thinking nature; could not impress the apprehensive sense. Inert matter could not be known or thought at all except as thus in some way actively engaging the thought, by positively impressing the sense. The only question is as to the relative amount or kind as compared with a proper mental or spiritual *datum*. And inasmuch as this character of energy is a fundamental condition to thought, one of the lowest

principles in regulating the division and classification of knowledge or science is to be found just here in the relative kind and amount of energy given to thought in its different apprehension of mind and matter. And as already intimated, the further specifications of science will be governed by the attributes that are discerned by thought in the diversity of *data* presented to it from the universe of the real and the true.

§ 196. But logical science supplies another most important principle in regulating all legitimate knowledge of the real universe as its divers parts are taken up for study. Whatever part is thus taken up into thought is an organic part; and can never be fully thought or known merely from its own intrinsic properties. It can never be separated in full and true knowledge from its related parts. As well might a limb of the body be fully and truly known independently of all relation to the body and to other members. The part by which it was joined to the organism must necessarily show more or less of the character of the related organs, and so of the whole organism; must show thus more or less beyond what attaches to the intrinsic nature of the particular member that has been exscinded. All that is thus revealed is a part of the *datum*. In accepting it as such, thought does not transcend its bounds. If a member, there must have been a body; there must have been a junction; the faces of the parts, organically joined, but now

separated, must correspond. If, for instance, the part be a finger, there must be a hand to which it was joined, an arm, a body. If the finger was real, so must the hand be real, for a continuity of the real was separated in the dismemberment. If it was fleshly, as the disruption was of flesh the hand must to some extent at least be fleshly. It is clearly beyond all reach of possibility to suppose that in an organic part all that is presented to thought is comprehended in the intrinsic attributes of the part itself. The nature of thought warrants and for the ends of perfect knowledge or science necessitates the admission, at the same time and together with the part, of something of its organic relationship, of something consequently beyond all intrinsic properties.

Logical science, it is thus shown, legitimates a vast and most important kind of knowledge. It recognizes and authenticates as true science that which may be founded on the organic relationships between the different parts of the real universe presented to human thought. The principle is that which validates that most important and most interesting department of knowledge of recent origin, embracing the particular sciences of Comparative Philology, Comparative Anatomy, Comparative Biology, etc. It is a principle acted on continually in the most familiar experiences of life.

§ 197. We have here to apply this principle in

order to complete our survey of the relations of thought and of logic as the science of thought to the contents of the real universe, to the great questions as to the genesis and the nature and validity of our notions of time and space. The logical principle more predominantly concerned here is that the part is organically related to the whole ; that so a given part involves the whole of which it is a part ; that so far as an organic part, any part given to thought necessarily brings in the whole so far as organically the same. The attribute of reality, as a fundamental attribute and an essential condition of all thought and knowledge, enters into the inmost essence of every object that can engage human thought ; and if given in any object accepted as part, involves the reality of that of which it is a part.

We have found that any part of the real given to thought may be thought as a substance. To all substance, given to thought, mediately at least through the outer senses, belongs the attribute of position and extension ; it is here or it is there ; and it extends from this position to that. No attribute is more unquestionably given to the apprehensive sense directed toward the outer world than this. The attribute is certainly everywhere in the human consciousness. Thought is utterly incompetent to create it ; and imagination is as incompetent. It is a true presentation from without ; a veritable part of the *datum*, recognized by thought as part. But this char-

acter of extension discerned in every material object is real as the object itself. It discloses also the marks of a part severed from a kindred part, from a whole of which it is a constituent. The particular extension implies thus a whole of extension which is as real as the particular position and extension. And this is *space*. Space, as the organic whole of particular extension, is thus given us as a real from without in every mental apprehension of a material body. The genesis is clear. The notion is legitimate. It is in accordance with the fixed principles of thought.

The Kantian argument is futile :—that if we abstract all the attributes given in our notion of a material body we can never abstract that of space, and therefore we must conclude it was not given in the object and must have existed independently of it and *àpriori* to it. You may in like manner abstract all the parts of a limb severed from a living body, and so conclude as legitimately that the giving to thought of the limb could not have given also to some extent the body to which it belonged, and end with the conclusion that the supposition that there was a body to which it belonged is invalid.

Space is thus a portion of the contents of the real. It is real; is brought to thought in the attributes of any given material object. It is a whole. It is not finite, for no limit is given to thought. It is not, however, therefore infinite, as we have seen.

The genesis of the notion of time is in exact analogy. A causal part of the real universe is given to the apprehensive sense in thought. An energy, a power, a force, in motion is discerned. It begins, continues, ends, or rather, and more accurately, thought severs a portion of the flux of the real universe from the rest, and discerns a beginning, progress, end; it discerns in other words the character of duration. But this character of duration is discerned as a part; it is an organic part of a comprehensive whole. And in this discerned part is also discerned, by the necessities of organic relationship, the whole, so far as essential nature is concerned. This whole is *time*. It is a real. It is as such given to thought and is not created out of nothing by thought, and was not created in thought at its origin. It is a whole, comprehending all particular durations. It is not finite, but, as we have seen, not therefore infinite.

§ 198. Science has been summarily defined as *systematized knowledge*. This is far more and other than generalized, or classified, or co-ordinated knowledge. It is knowledge or thought wrought into a system. A system comprises three essential constituents: unity, totality, and harmony or congruence. As all thought or knowledge is essentially attribution, this threefold nature of systematic knowledge implies that there be one subject which shall embrace every attribute presented as constitutive of the science;

that all the essential attributes be given; and that these attributes be presented organically, that is, in their harmonious and orderly relationships.

§ 199. It is apparent from what logical science has taught us concerning thought and its object, that there may be as many conceivable sciences as there are parts of the universe of the true that can be apprehended and bounded and viewed in their several constituent parts. Indeed there are conceivably manifold more than these; for the enumeration and classification of the sciences may proceed on either one of several different principles of division. An absolute pantology is indeed impossible to human thought; for the totality requisite in a system can never be attained since, as has been seen, the universe of the true is without bounds. Gradations of sciences in respect of comprehension are conceivable to an indefinite extent along any one line of division. Cross-divisions, moreover, are possible everywhere, as also sciences of a mixed character. The number of sciences possible to thought is thus unlimited.

This department of logical doctrine which treats of the extrinsic relations of thought, especially those of thought to its object, might not ineptly bear the name of *Heuretic Logic*, *Logica Heuretica*, which has been in a more indefinite way applied to its exposition of the rules of invention or discovery. But associated under this

general head of the extrinsic relations of thought which expounds the nature and forms, the extent and the content of the true as the objective real which may be found and acted upon in thought, is another department of logic which may with like propriety be denominated *Architectonic Logic*, *Logica Architectonica*, in which the science appears as constructive or building, unfolding the principles and forms in and by which the true, as found, is to be wrought into systematic or organic wholes—into sciences. This view of logic, as science of the sciences, *Scientia Scientiarum*, the consideration of the relations of thought to systematized truth, constitutes properly the second division of this part of logical science.

§ 200. Principles regulating the division and the classification of the sciences may be found in either of the three conceivable elements that determine the construction of thought into systems;—the nature of thought itself as the efficient factor in the construction; the nature of the true as the material to be wrought into scientific form; and the character of the end or design for which under the guidance of its rational nature thought must move in its constructive work. These may be styled the *logical*, the *objective*, and the *telic* principles of scientific classification.

I. Logical Classification of Sciences.

§ 201. The logical grounds of scientific enumeration and distribution are given at once in the categories of pure thought. I. On the category of identity is founded the science known as ontology, the science of knowledge and of being or the fundamental science of the true as the real in thought. It is otherwise denominated Fundamental Philosophy; Metaphysics; Fundamental Logic. It builds on logic as its immediate and conditional basis.

II. On the category of quantity is founded the classification of sciences into the analytic and synthetic, with which correspond to some degree what are called the deductive and the inductive sciences. Analytic science properly turns its view upon the attribute or the predicate concept, and separates it into its component parts, exhibiting them in their relative order and congruence. It also legitimately deals with the generic concept, separating it into its logical constituents. Or abstracting from its investiture of thought in which the real has become clothed in becoming the true—the real itself in its simple purity and simplicity—it resolves the collective multitude into the numerical constituents or the homogeneous mass into its component parts. Synthetic science, on the other hand, puts together component parts of like nature according to their true

order and gives as its results generalized facts, comprehensive truths, collections of the many, or masses of the homogeneous.

III. On the category of modality or necessity is grounded the distinction of the necessary or apodictic, and the empirical sciences.

II. Objective Classification of Sciences.

§ 202. The most fundamental principle of scientific classification furnished by the matter of thought or its *datum* is from its most essential characteristic—the real as active. As has been shown (§ 191), all matter that can engage the activity of thought must impress it by virtue of an inherent energy or force in it. The diversity of the forms of this energy, by and through which all matter of thought interacts with it, must accordingly furnish the broadest and most fundamental principle for distinguishing and classifying the divers sciences so far as they are distinguished by the subject-matter of which they respectively treat. The first and deepest distinction that apprehensive thought can make here is that which comes from the observed source of the *datum* as from within, that is, from the mind itself, or from without. We have thus the fundamental division into the science of the self, the ego, the self-consciousness, and that of the not-self, the non-ego. But this last named energy, that engaging thought from without the conscious self, is dis-

cerned in some instances to be of like nature as the first named, the energy of the proper self; their natures or attributes are identical. The science of the ego is the same as the science of the non-ego to this extent. We have then the first grand division of science into—I. The science of the inner world, the realm of the conscious, the science of mind, of soul or spirit, by whatever name signified; and embracing both the ego itself and also the non-ego so far as acting with the same kind of energy; and II. The science of the outer world.

The first division embraces the science of Psychology in its broadest import, presenting the threefold distinction of energy in the threefold function of intelligence, sensibility and imagination, and will. Subdivisions here are the sciences of these threefold functions in their relation to their respective objects—the true, the beautiful, and the good, known as the sciences of Logic, Æsthetics, and Ethics. Still more subordinately within the realm of logical science lies the broad fields of quantitative thought, the mathematical sciences of number and magnitude—Arithmetic and Geometry with their respective ramifications.

The second division embraces two widely distinguishable classes of science, distinguished fundamentally by the character of the attribute, extrinsic or intrinsic, apprehended in the *datum*. The first-class embraces the sciences of Time and Space. As in these realities the quantitative char-

acteristics of thought first realize themselves the mathematical sciences may with no considerable impropriety or hazard to truth be ranked with or under these sciences. The other class embraces the properly physical sciences. They distribute themselves at once into the two classes (1) of Organic, the sciences of the real manifesting the presence of an energy from within characterizing the manifestations to thought, of the realities determined by the vital force, and the comprehensive biological sciences including biology proper, botany, zoology, etc.; and (2) Inorganic, the sciences of realities manifesting divers forms of energy or force acting through them rather than from within, transmitted rather than emitted. The subdivisions are the familiar physical sciences, multiplied and crossed in character indefinitely, by the application of other principles of classification.

III. Telic Classification of Sciences.

§ 203. The distribution of the sciences in reference to the specific end or object which is proposed in the construction of them gives the familiar classes of (1) the Sciences proper, and (2) the Arts. Science proper seeks truth for its own sake—pure knowledge; art seeks an object beyond mere knowledge—skill or power inspired, guided, trained by and in intelligence.

SPECIAL RELATIONSHIP OF THOUGHT TO LANGUAGE.

§ 204. Besides the relation which logic bears to the science of language or grammatical science common to other sciences as prescribing to them the regulative principles and methods of construction, it bears a peculiar relation to it from the fact that language is the proper embodiment of thought. Language proceeds from thought as its source and origin. It is produced in the natural outworking of thought and for the purposes of thought. It is the proper body of thought. Grammatical science can therefore be reached only through the science of thought. The nature of language can be understood only as the nature of thought is understood. The members of thought determine the members of its body in language; the forms of language are but the outer forms of thought. What the parts of speech must be, what their significancy and function, can be determined only by a knowledge of what the embodiment of thought must be in order to be true and subservient to the uses of thought.

The primal unit of thought—the judgment—determines thus the primal unit of language—the sentence. The trimembral constitution of the judgment—the subject, copula, and attribute—determines the corresponding essential character of the grammatical sentence—the subject, the asser-

tive element, and the predicate. There is no sentence, no embodied thought conceivable without these three elements all expressed or implied. The presence of the three makes out a complete sentence. The modifications of these essential constituents, with these essentials, make up language. The significance of the different forms, primitive or modified, of language is to be seen in the different primitive or modified forms of thought. The grammatical *noun* is thus the speech-form of the subject in the judgment, and can never be explained but as such. Its divisions into the proper noun and the common noun are the logical divisions of individual and generic subjects. The mass noun and the collective noun express logical subjects in which the thought investiture is abstracted from the real. The distinctions of nouns in respect to number, sex, and speech itself as essentially communicative, are such as the distinctions of thought require for their expression. The received definition of the grammatical personal pronoun as "a word used instead of a noun" is utterly inane. The pronominal distinction is the natural threefold distinction of subject-words or nouns in language regarded as communicative into words denoting *speaker*, *spoken to*, and *spoken of*. The adjective, logic teaches us, is the normal embodiment in language of the attribute, but specially of the attribute of property. The other attributes of thought, those of action and relation,

appear for the most part in the verb. In the progressive development of language these attributes appear combined with the assertive element. It is this combination which characterizes the common grammatical verb. The leading modifications of the attribute are determined by the character of the quantity as recognized in logical sciences. The two generic kinds of adjectives, definitives and essentials, are the different expressions of the two kinds of logical quantity. The so-called degrees of grammatical comparison also are determined by the different logical relations to be expressed, whether to the part or to the whole: if to the part, the so-called *comparative* is used; if to the whole, the *superlative;* the unmodified in this respect being the *positive.* That the superlative necessarily expresses a higher degree of quality than the comparative, as commonly taught, is entirely erroneous: Methuselah is not declared to be of a greater age—to possess a higher degree of that attribute of age—when he is said to be *the oldest of men* than when he is said to be *older than all other men.* In like manner the exposition of the verb as the expression of the assertive element, of its moods as expressing the logical distinctions of thought as to its character of modality—of necessity and contingency—is impossible save in the light of logical science (§ 43). The distinction of the forms and connections of words determined by the thought or the life-principle of

language from those which belong only to the word as the body of thought, all important as it is to any complete and assumed science of language, can of course be known only from the science of thought. Indeed, to speak summarily here, the intelligent and confident use of language and the ready and sure interpretation of it are attainments that can be made only by the help and guidance of logical science. Grammar goes blind, limping, stumbling, except as under the lead and support of logic.

APPENDIX.

EXERCISES.

NO. I. EXERCISES IN THE PARTS OF A JUDGMENT. (PART I. C. iii. §§ 23–33.)

Point out the words which severally express the subject, the predicate, and the copula in the following propositions:—

1. The spirit is deathless. 2. The spirit never dies. 3. The earth is heavy. 4. The earth gravitates. 5. Great was the wonder. 6. Grandly was the undertaking effected. 7. Before her steps walketh prudence. 8. Knowledge is the pearl of price. 9. How sublime the spectacle! 10. There is no rain in that region. 11. It is not material whether he go or stay. 12. It rains. 13. It is cloudy. 14. To think is instructive. 15. The year passes. 16. Where he now resides is unknown. 17. Why he removed has not been explained. 18. It is not known why he removed. 19. The sun is up. 20. There is not a cloud in the sky. 21. It is a sunny day. 22. He of necessity remains weak who never exerts himself. 23. Who shunneth disease hooteth at remedies. 24. It is cruelty to load a falling man. 25. Nothing is harmless that is mistaken for a virtue. 26. It is a poor center of a man's actions, himself. 27. Life every man holds dear. 28. Though gods they were they died as men. 29. There be none of the affections which have been noted to fascinate or bewitch, but love and envy. 30. Handsome is that handsome does. 31. He denies himself who asks what it is impossible to grant. 32. To err is human: to for-

give divine. 33. The city for wealth; the country for health. 34. There is no joy without alloy. 35. Easy learned is soon forgotten. 36. All is not gold that glitters. 37. There is no courage but in innocence. 38. Many are the deserving men who are unfortunate. 39. So false is faction. 40. One aid to health is exercise.

NO. II. EXERCISES IN THE KINDS OF JUDGMENT. (§§ 34–46.)

1. *Name the several judgments that follow :—*

1. Quadrupeds are vertebrate. 2. No insect is vertebrate. 3. Every mistake is not culpable. 4. Not one escaped. 5. Animals are vertebrate or invertebrate. 6. A proud man is necessarily a self-deceived man. 7. Truth may languish but can never perish. 8. If the soul is incorruptible it is undying. 9. No news is good news. 10. All mad men are unwise. 11. All unwise men are not mad. 12. Right is right. 13. Matter is either solid, or liquid, or gaseous. 14. It is never too late to learn. 15. A rolling stone gathers no moss. 16. Where there is no law there is no trespass. 17. They are at hand to parley or to fight. 18. No yesterday returns. 19. No receiver, no thief. 20. It is not the many oaths that make the truth. 21. Invertebrate animals are either with longitudinal axis or not. 22. All invertebrates without longitudinal axis are radiates. 23. If an invertebrate has a longitudinal axis it is either a mollusk or an articulate. 24. None but the brave deserve the fair. 25. No alchemy is equal to saving. 26. High regions are never without clouds. 27. It is not in might to authorize oppression. 28. Not being rich is not always an evil. 29. Beggars should not be choosers. 30. No one is always happy. 31. Forbearance is no acquittance. 32. The principal acids are oxacids or hydroacids. 33. Not many of the metals are brittle. 34. If ignorance is bliss, 'tis folly to be wise. 35. Fame is no plant that grows on mortal soil. 36. If the aye-

aye be a bird, it is either carinate or struthious. 37. He is either a fool or a knave. 38. If he is not a fool, he is a knave. 39. Money is money in these times. 40. A triangle is a three-sided figure.

2. *Point out the Negative judgments in the preceding list of propositions.*

3. *Point out the Disjunctive judgments in the list.*

4. *Point out the Hypothetical judgments.*

5. *Point out the Modal judgments.*

6. *Point out the Identical judgments.*

NO. III. EXERCISES IN CONCEPTS. (§§ 52–65.)

1. *Point out the single concepts in the following propositions :—*

1. Sirius twinkles. 2. The horse is fourfooted. 3. Man is rational. 4. Birds are twofooted and winged. 5. The sun is bright. 6. Animals are of four divisions. 7. All living things are organic. 8. Life is vegetative, animal, or spiritual. 9. Spirit is immaterial. 10. All men are mortal. 11. Mortality is universal among men. 12. Every mistake is not a proof of ignorance. 13. The wicked shall fall by his own wickedness. 14. A regiment consists of two battalions. 15. Ice is cold. 16. Time abates grief. 17. Tyranny is irresponsible power. 18. No news is good news. 19. Good orators are not always good statesmen. 20. Britain is an island. 21. A man is a man. 22. The wages of sin is death.

2. *Point out the simple concepts in the foregoing list.*

3. *Point out the generic concepts.*

4. *Point out the composite predicate-concepts.*

NO. IV. EXERCISES IN IMMEDIATE REASONINGS. (§§ 70–77.)

1. *Point out in the following list the Immediate Reasonings by Conversion.*

2. *Point out those by Restriction, whether Modal or Quantitative.*

3. *Point out those by Disjunction.*

4. *Point out those by Transference.*

5. *Point out those by Composition.*

Or, *name each reasoning on the list in order.*

1. All ruminants are herbivorous mammals that chew the cud; therefore all herbivorous mammals that chew the cud are ruminants.

2. All human action is voluntary or involuntary; therefore all that is not involuntary is voluntary.

3. The planets all reflect their light; therefore, the inferior planets, Mercury and Venus, are not self-luminous.

4. All birds are feathered bipeds; therefore all feathered bipeds are birds.

5. Man necessarily acts morally; therefore he is a moral agent.

6. Every animal has a circulatory system; every vegetable has a circulatory system; therefore all organic being has a circulatory system.

7. Equi-lateral triangles are equi-angular; therefore equi-angular triangles are equi-lateral.

8. Vertebrates are mammals or birds; therefore birds are not mammals.

9. The anemone has no proper corolla; therefore it is apetalous.

10. Planets are wandering stars; therefore wandering stars are planets.

11. A shrub is a perennial plant with woody stems; therefore perennial plants with woody stems are shrubs.

12. No mammals are oviparous; therefore no oviparous animals are mammals.

13. Cold-blooded vertebrates are either reptiles or fishes; therefore no fish is a reptile.

14. The monkey tribe of the old world are foot-handed; the monkey tribe of the new world are foot-handed, as are

also those of Madagascar: therefore all of the monkey tribe are foot-handed.

NO. V. EXERCISES IN CATEGORICAL REASONINGS. (§§ 78–80.)

1. *Point out the deductive reasonings in the following examples.*

2. *Point out the inductive reasonings in the following examples.*

Or *name each reasoning in order*.

1. The planets move in elliptic orbits; therefore the asteroid last discovered moves in an ellipse.

2. The tree last year bore a reddish sweet apple; therefore its fruit will be the same the coming season.

3. All mammals are warm-blooded; therefore the whale is warm-blooded.

4. These captured zebras are solid-hoofed; therefore wild zebras are solid-hoofed.

5. The giraffes in the Paris Garden of Plants and the London Zoölogical Gardens are timid, gentle, and docile and browse on twigs and leaves; therefore all giraffes have the same characters.

6. "If the Catskill sandstone be the base of the carboniferous system in America, the old red sandstone of Scotland, which has been identified in age, must be the base of the carboniferous system in that country."—*Winchell.*

7. The ichthyosaurus had the teeth of a crocodile; therefore it was carnivorous.

8. "There are many cases, recorded by medical observers, of persons losing their sight, who have also lost the faculty of representing the images of visible objects. Similar cases are recorded in regard to the deaf. These facts make it, I think, more than probable that there are as many organs of imagination as there are organs of sense."—*Hamilton.*

9. Herbivorous animals have grinding teeth and a complicated digestive system; so it is accordingly with the giraffe.

NO. VI. EXERCISES IN DEDUCTIVE REASONINGS. (§§ 81-86.)

1. *Name the minor term, the major term, and the middle term in each of the following reasonings.*

2. *Name in regular syllogistic order the sumption, the subsumption, and the conclusion in each of the following reasonings, supplying the wanting premise when necessary.*

NOTE.—The proper way of proceeding is first to recognize in thought the conclusion; its subject will be the minor term, and its predicate will be the major term. The middle term will be found in the given antecedent. Then the sumption can be formed of the middle term as subject and the major term as predicate; and the subsumption, of the minor term as subject and the middle term as predicate. The propositions should be arranged in the order of sumption, subsumption, and conclusion.

3. *Name each proposition in each syllogism thus regularly constructed as A, E, I, or O; and state what premises conclude in A; what in E; what in I; what in O.*

1. All planetary bodies revolve in elliptic orbits; the asteroids are planetary bodies; therefore, they revolve in elliptic orbits.

2. M is W; P is M; therefore P is W.

3. The planetary body just discovered has a very eccentric orbit; for it is an asteroid; and all asteroids have very eccentric orbits.

4. P is W, for P is M; and M is W.

5. Venus reflects its light; therefore Venus has phases like the moon.

6. P is M; therefore P is W.

7. All satellites revolve on their axes in the same direction as their primaries; therefore the moon revolves in the same direction as the earth.

8. M is W; therefore P is W.

9. The sun revolves on its axis from west to east; for it is the earth's primary.

10. P is W; for P is M.

11. The planetary body just discovered revolves between the orbits of Mars and Jupiter; for all the asteroids are in that part of the heavens.

12. P is W; for M is W.

13. Educated among savages, he could not be expected to know the customs of polite society.

14. P being M is W.

15. As almost all the organs of the body have a known use, the spleen must have some use.

16. M being W, P is W.

17. Unpleasant things are not always injurious; for afflictions are often salutary.

18. Solon must be considered a wise legislator, seeing that he adapted his laws to the temper of the Athenians.

19. I think; therefore I am.

20. Corporations have no souls; for they are created by the king, and none can create souls but God.

NO. VII. EXERCISES IN INDUCTIVE REASONINGS. (§§ 87–88.)

1. *Name the minor term, the major term, and the middle term in each of the following reasonings.*

2. *Name in regular syllogistic order the sumption, the subsumption, and the conclusion in each of the following reasonings, supplying the wanting premise where necessary.*

NOTE.—The proper way of proceeding is first to recognize the conclusion: then with its predicate as major and the third or middle term, which is here a complementary of the minor term, to be found in the antecedent, construct the sumption if not given, and with the subject of the conclusion as minor term and the third or middle (complementary) term, construct the subsumption if not given; and subjoin the conclusion.

1. This magnetized iron attracts these iron filings. This

other piece of iron is magnetized; therefore it will attract the filings.

2. P is W; C is P; therefore C is W.

3. This iron is magnetic; it will therefore attract iron filings.

4. C is P; therefore C is W.

5. Magnetized iron attracts iron filings; therefore this piece of magnetized iron will attract iron filings.

6. P is W; therefore C is W.

7. This piece of iron will attract iron filings, for it is magnetic.

8. C is W; for C is P.

9. The amethyst is a silicate, for it belongs to the same kind of minerals as chalcedony which is a silicate.

10. The sapphire belongs to the same class of minerals as common emery, and is therefore an oxide of alumina.

11. Carbon is fusible in the compound blowpipe; so consequently is the diamond.

12. That grass field must be wet, for the grass here is covered with a heavy dew.

13. The water pitcher is covered with moisture; we may therefore expect to-night, if it is clear, a heavy dew.

14. The expenditure of muscular force represents a definite combustion of the bodily substance, as we may infer from the use of steam in the locomotive.

15. The earth's moon revolves like its primary from west to east; therefore the fourth moon of Jupiter revolves like its primary from west to east.

16. The disturbances of the motions of Uranus proved the existence of an undiscovered planet not within the known planetary system; Mercury suffers like disturbances to those of Uranus; therefore there is an undiscovered planet not within the known planetary system.

17. The circulation of the sap is in part due to capillary attraction, for that of the blood is due in part to that cause, and the circulatory system in plants is identical with that in animals.

NO. VIII. EXERCISES IN HYPOTHETICAL REASONINGS. (§§ 91–95.)

1. *Name in regular syllogistic order the sumption, the subsumption, and the conclusion in the following reasonings, supplying the wanting proposition when necessary.*

2. *Name the conditioning judgment and identify the reasoning accordingly as ponent or tollent.*

3. *Point out the dilemmas in the list.*

1. If virtue must under a perfect government in the end be rewarded, then virtuous men will be happy; but virtue under such a government must eventually receive its due reward; therefore, virtuous men will be happy.

2. If W is, P is; but W is; therefore P is.

3. If virtue must receive its due reward, then virtuous men will be happy; but virtuous men are not all happy in the present life; therefore virtue does not receive in the present life its due reward.

4. If W is, P is; but P is not; therefore W is not.

5. No rain has fallen, for if there had, it would be wet.

6. W is not, for if W is, P is.

7. The ground is wet, for if it has rained it must be.

8. P is, for if W is, P is.

9. If the moon has no atmosphere there cannot be any twilight; but it has no atmosphere; therefore there is no twilight on the surface of the moon.

10. If there be no future life, then either virtue receives its due reward in the present world or there is no perfect government administered over men; but neither of these alternatives is true; therefore there is a future life.

11. If natural selection lengthened the neck of the giraffe to enable it to reach the foliage of trees for preservation in severe droughts, it should also have lengthened the necks of other ungulates in like circumstances; but it has so favored none but the giraffe; therefore, the principle of natural selec-

tion cannot be accepted as accounting for the long neck of the giraffe.

12. Ye are not the true children of Abraham; for if ye were, ye would do the works of Abraham.

13. The solar heat cannot be the effect of combustion; for if it were, its supply could last but for a few thousand years at most.

14. There is no improbability in the supposition that vegetable germs may have been preserved to this time in the deposits of the glacial age; for if muscular fiber so deposited can survive, certainly vegetable tissues may; and yet we know that dogs and bears in Siberia eagerly devour the remains of the extinct mammoth.

NO. IX. EXERCISES IN DISJUNCTIVE REASONINGS. (§§ 96–97.)

1. *Name in regular syllogistic order, the sumption, the subsumption, and the conclusion in the following reasonings, supplying the wanting proposition when necessary.*

2. *Identify the reasoning as Ponent or Tollent.*

1. A fungus is either plant or animal; but it is not animal; therefore it is plant.

2. A is B or is C; but it is not C; therefore it is B.

3. The world is the product of intelligence or of chance; but it is the work of intelligence; therefore it is not the product of chance.

4. A is either B or C; but it is B; therefore it is not C.

5. Light must be a mode of motion; for either it is a mode of motion, or it is material; and it is not material.

6. A is B; for A is B, or is C; and it is not C.

7. Either the fourth commandment of the decalogue is not obligatory, or we are bound to set apart one day in seven for religious duties; but the fourth commandment is obligatory.

8. Either virtue is not voluntary or vice is voluntary; but virtue is voluntary.

9. He must be a knave; for he is evidently no fool, and he is either a knave or a fool.

10. The footprints in the sandstone of the Connecticut river valley are either those of reptiles or birds; but they must be the tracks of reptiles, for this was the age of Saurians: they are not therefore the tracks of birds.

11. Real induction, or the induction of common life is an operation of thought or it is not; if it is not an operation of thought, then it leads to no truth and it is utterly worthless; if it is an operation of thought, then it is within the scope of that science which explains the nature and expounds the laws of thought.

NO. X. EXERCISES IN IRREGULAR REASONINGS. (§§ 98–102.)

1. *Resolve the complex reasonings in the following list into their constituent reasonings and give to each its proper name.*

2. *Change the regressive reasonings into progressive.*

1. All just acts are expedient; therefore some unjust acts are inexpedient.

2. No unhappy persons are righteous; for all the righteous are happy.

3. No war is long popular, for every war increases taxation; and the popularity of anything that touches our pockets is short-lived.

4. Some deserve to be imitated who are nevertheless fools; for whoever speaks the truth deserves to be imitated.

5. The preservation of society requires the adoption of a common measure, for the preservation of society requires exchange; and this requires equitable valuation of property; and this requires the adoption of a common measure.

6. The English are a brave people; a brave people are

free ; a free people are happy ; therefore the English are happy.

7. A science is either pure or inductive ; a pure science is either mathematical or logical ; but astrology is neither mathematical nor logical, nor inductive ; therefore, it is not a science.

8. The bat is a mammal ; for it is either a mammal or a bird, and it is not a bird, for it is not oviparous.

9. If we have an intellectual nature in common, the reason is also common ; if this is so, its essential attribute of law-giving is common ; if this is so, there is a common law ; if this is so, we are fellow-subjects ; if this is so, we are members of a political community ; if this is so, all mankind make up a kind of state.

NO. XI. EXERCISES IN THE METHOD OF JUDGMENTS. (§§ 114–123.)

I. OBJECTIVE LAWS.—*Point out the errors in the following propositions in respect to unity and congruence, adequateness, and accuracy.*

1. Triangles are acute, obtuse, equilateral, or scalene.

2. We find in every stable government either an hereditary monarchy, or an enlightened and virtuous democracy, or misrule and sedition.

3. The domain of the United States comprehends both states and territorial organizations.

4. Triangles are right-angled, acute-angled, or spherical.

5. The planets, the comets, the asteroids, and the moon revolve around the sun in elliptic orbits.

6. Sensation, perception, imagination, memory, and judgment constitute the phenomena of the human mind.

7. All that makes up a virtuous act is a knowledge of duty and a disposition to do it.

8. The elements of a true civilization are a wise and just polity, a general intelligence, and an æsthetic culture.

9. Inductive sciences are those which are founded in actual observation; while deductive sciences proceed from mere assumptions of the human mind.

10. Inductive sciences are not comprehended within the sphere of the discursive intelligence which limits itself to the relations of the individual to the universal.

11. An acute-angled triangle is one which has an acute angle.

12. The ponderable bodies embrace solids and fluids, which with the imponderable bodies make up the entire class of material bodies.

II. DISJUNCTIVE PROPOSITIONS.—1. *Distinguish in the following propositions those that do not disjoin in contradictory opposition.*

2. *Test such as have more than two disjunct members by reducing them to others in true contradictory opposition.*

3. *Point out such as are faulty in any respect—in matter, in thought, or in expression.*

Or *describe in order each proposition in respect to its disjunctive character.*

1. Trees are either evergreen or deciduous.

2. In morals we must advance or fall back.

3. A currency is good as it is safe, or as it is elastic, expanding and contracting with the wants of trade, or as it is of convenient denominations.

4. A good horse is valuable for speed or for strength.

5. Plants are annuals, biennials, or perennials.

6. Invertebrates are either radiates, or mollusks, or articulates.

7. Plants are either esculents, drugs, or poisons.

8. Virtue tends to procure us either the esteem of mankind, or the favor of God.

9. A successful author must be either very industrious or very talented.

10. Reason is her being, discursive or intuitive.

11. The cable was loosed or unloosed.

12. All life is rational, or sentient, or animal, or vegetable.

13. He is sitting, or lying, or standing.

14. Planets or their satellites include all the members of the solar system.

15. Herbs, shrubs, or trees embrace every vegetable growth.

III. HYPOTHETICAL PROPOSITIONS.—1. *Reduce the hypothetical to the propositional form of stating the terms.*

2. *Distinguish the Inductive propositions from the Deductive.*

1. If the ground is wet, rain has fallen.

2. If pain is severe, it will be brief.

3. If the will of Henry VIII. was valid, James I. had no legal title to the throne of England.

4. If the flight of birds to the north has begun, vegetation must have started where they wintered.

5. If one angle of the triangle is right, the others are to each other complementary parts of a right angle.

6. If Mars has an atmosphere, the land as distinguished from the water must appear greenish, and the polar regions white from the presence of the snow.

7. If this fossil had horns, it was a ruminant.

8. If a stone tends to fall to the ground by force of gravitation, then the moon tends to fall to the earth.

9. If an ordinary pump will lift water only thirty-three feet in the valley, it will not lift it so high on the top of the mountain.

IV. VERBAL LAWS.—*Correct the faults in the following propositions, whether pertaining to the matter, the thought, or the verbal expression.*

1. All words which are signs of complex ideas furnish matter of mistake.

2. Fortunes are made by industry or frugality.

3. All that glitters is not gold.

4. Tropical climates only produce rice, bananas, and spices.

5. No trifling business will enrich a man.

6. All the trees in the park make a thick shade.

7. All the plays of Shakespeare cannot be read in a day.

NO. XII. EXERCISES IN THE METHOD OF CONCEPTS. (§§ 124–137.)

1. *Form gradations of classes with an individual* man *as subject and with the attribute of* habitation *as the base ;*

With the attribute of age ;

With the attribute of condition ;

With the attribute of color ;

With the attribute of social relation.

Generalize from an individual plant *as subject ;*

From an individual animal ;

From an individual virtue *as* gratitude.

2. *Amplify by determination the following attributes, distinguishing essential from relative attributes.*

Being, Human, Organic, Endogenous, Vertebrate, Intelligent, Grateful, Social.

3. *Define the following terms :*

1. The Sun, a Plant, a Man, a Cedar, a Rose, a Virtue, a Monarchy, a Poem, a Statue, a Habit, a Law.

2. Bright, Curvilinear, Fragrant, Skill, Liquid, Organic, Credulous, True, Perspicuous, Rash, Weary, Courteous, Temperate.

4. *Divide the following concepts :*

Being, Mammals, Man, Carnivorous Quadrupeds, Ruminants, Birds, Reptiles, Radiates, Flowering Plants, Apetalous Plants, The Rose, The Honeysuckle, Rocks, Geological Ages, Physical Forces, Colors, Governments, Arts, Sciences, Virtues.

5. *Analyze by Partition the following concepts :*

Organic, Oviparous, Winged, Carnivorous, Batrachian, Mental, Moral, Civil, Patriotic, Artistic.

NO. XIII. EXERCISES IN METHOD OF REASONINGS. (§§ 138–160.)

Verify the following reasonings, indicating in such as are unsound the seat of the fallacy.

1. All just acts are expedient; therefore all expedient acts are just.

2. All winged animals are not birds; therefore all birds are not winged.

3. All beings are either resolute or are not resolute; but irrational beings are not resolute; therefore they are irresolute.

4. A democracy is not subject to violent revolutions, for the great mass of the people have all political power in their own hands.

5. Every one desires virtue, because every one desires happiness.

6. The exportation of gold is to be deprecated, for it is the abstraction of so much wealth.

7. All ruminants of the present age have cloven feet and are horned; therefore this cloven-footed fossil must have belonged to a horned ruminant.

8. The barometer has fallen; therefore there will be rain.

9. There will be a change in the weather; for the moon quarters to-day.

10. The blood is propelled by capillary attraction; for fluids in capillary tubes are raised about the sides of the tubes.

11. Potassium is easily fusible and volatile at a high temperature; rubidium and lithium must be so likewise, since they are also alkaline metals.

12. No carnivorous animals have four stomachs; all ruminants have four stomachs; therefore, no ruminants are carnivorous.

13. All plants contain cellular tissue; no animals are plants; therefore, no animals contain cellular tissue.

14. A man that hath no virtue in himself ever envieth virtue in others; for men's minds will either feed upon their own good or upon others' evil.

15. Humanity is a moral virtue; the study of polite letters is humanity; therefore the study of polite letters is a moral virtue.

16. Of two evils the less is to be preferred; occasional turbulence, therefore, being a less evil than rigid despotism, is to be preferred to it.

17. Truth is not duty; a revelation of duty should not therefore be expected to be true.

18. Miracles were never performed, for the simple reason that they are impossibilities.

19. Have you left off beating your father? No. Then you are an execrable wretch.

20. Have you left off slandering your mother? Yes. Then what an execrable wretch you were.

21. Animal food may be dispensed with, for the Brahmins live without it; and vegetable food may be dispensed with, for the Esquimaux live without it, therefore all food may be dispensed with.

22. If we never find skins except as the teguments of animals, we may safely conclude that animals cannot exist without skins.

23. If color cannot exist by itself, it follows that neither can anything that is colored exist without color.

24. If language without thought is unreal, thought without language must also be so.

25. Electricity is not the real agent by which the nerves act upon the muscles; for electricity is transmissible along a compressed nervous trunk which has lost from the compression all its power to move a muscle.

26. Gambling is a violation of the tenth commandment, for it involves a desire to gain by another's loss.

27. Government is a trust, for it is not a property.

28. Death must be agreeable, for it is a change, and all change is agreeable.

29. The ancient Greeks produced the greatest master-pieces of eloquence and philosophy; the Lacedemonians were ancient Greeks; therefore they produced the greatest master-pieces of eloquence and philosophy.

30. He who calls you a man speaks truly; he who calls you a fool calls you a man; therefore he who calls you a fool speaks truly.

31. Who is most hungry eats most; who eats least is most hungry; therefore who eats least eats most.

32. He who necessarily goes or stays is not a free agent; you must necessarily go or stay; therefore you are not a free agent.

33. Substances are either solid, liquid, or gaseous; the mind is a substance; it is therefore a solid, a liquid, or a gas.

34. If the fourth commandment is obligatory we are bound to set apart one day in seven for religious purposes; but we are bound to set apart one day in seven for such purposes; therefore the fourth commandment is obligatory.

35. "A concrete is the name of a thing;" "adjectives are concrete, not abstract;" therefore adjectives are names of things.

36. There must be a Divine Providence, for if there were not, no human governments could long subsist, and history teaches us that various human governments have subsisted long.

37. If I denied the being of God, I should be impious; I do not deny the being of God; therefore I am not impious.

38. He is brave who conquers his passions; he who resists temptation conquers his passions; so that he who resists temptations is brave.

39. Intelligent beings are not fit subjects for experimental inquiry; for where there is intelligence, there is sensibility to pain, and we are not at liberty to inflict pain.

40. A true philosopher is independent of the caprices of fortune, for he places his chief happiness in moral and intellectual excellence.

41. It was to be expected that geology would be at first denied as hostile to religion, for astronomy was so denied at its first beginning.

42. If sharpers are present we ought not to gamble; but there are no sharpers present, therefore we ought to gamble.

43. Confidence in promises is essential to the intercourse of human life; for without it the greatest part of our conduct would proceed upon chance. But there could be no confidence in promises, if men were not obliged to perform them; the obligation, therefore, to perform promises is essential to the same ends and in the same degree.

44. *Horse* is a concrete; and a concrete is a physically existing thing; therefore *horse* is a physically existing thing. But *horse* is a general term, for it includes many individuals; therefore one general term at least is a physically existing thing, and the realists were consequently right in maintaining that our general conceptions are entities.

45. True virtue is disinterested, but motives ever appeal to our interest or to what concerns us; therefore to be influenced by motives is incompatible with true virtue.

46. A species is a class of organic beings having a common parentage; evolutionism supposes beings originating without parentage; it therefore rejects the existence of any proper species.

47. If deference to authority be the source of the principle of duty in the hearts of men, it would be either because such deference is found to be useful or because it is felt it should be deferred to; but the first alternative is rejected by the author of the theory, and the second makes the sense of duty its own source; therefore deference to authority is not the source of the principle of duty.

48. That which is utterly inconceivable or unthinkable can

receive no attribution, for to attribute is to conceive or think. The infinite Being is therefore not inconceivable, for to affirm this is to ascribe to Him this attribute.

49. Pangenesis maintains that all propagation is by gemmules which pervade every living organism, and are derived from every part of each similar organism in each less remote ancestor. In some families for generations the beard has been shaven; and hence no gemmules could have come from the beard to their offspring; therefore such families must become beardless.

50. In comparison the two objects compared must both be in the consciousness at the same time or the one present and the other not; but it is impossible that there be two objects at the same time in the consciousness, and equally impossible to think of an object not in the consciousness; on either alternative comparison is impossible.

51. "Either the *ego* which is supposed to determine or will the action is some state of consciousness, simple or composite, or it is not. If it is not some state of consciousness, it is something of which we are unconscious; something therefore that is unknown to us; something, therefore, of whose existence we neither have nor can have any evidence; something therefore which it is absurd to suppose existing."

52. Either the sky is some state of consciousness, or it is not. If it is not some state of consciousness, it is something which it is absurd to suppose existing.

53. "If every instinctive action is an adjustment of inner relations to outer relations, which it is impossible to deny; if every rational action is also an adjustment of inner relations to outer relations, which it is equally impossible to deny;" then "the common notion that there is a line of demarkation between reason and instinct has no foundation whatever in fact."

54. If I divide nothing by nothing I find the quotient to

be $7 - \frac{0}{0} = 7$; for the product of this quotient multiplied by the divisor equals the dividend—$7 \times 0 = 0$.

55. I build my philosophical system by assuming nothing, —0. I posit this by affirming nothing of it—*nothing is nothing*—0 = 0. I have now a real thought with nothing as subject, nothing as attribute. But I am equally competent to deny as to affirm: therefore I deny with nothing as subject: *nothing is not something*. I have now evolved from nothing not only real thought but also, by this potency of mere nothingness or pure negation, real being. But nothing can produce from nothing only nothing; therefore the real thought and real being which I have evolved from nothing is only nothing itself;—being and non-being are the same. Still farther nothing is infinite, for it is bounded by nothing; and yet it is finite because it is bounded by its opposite—something, since nothing is not something. Once more the finite and the infinite are the same, for they are each the attributes of nothing.

56. Space, according to the Berkeleian view, is no intellectual notion, nor yet is it perceived by any of the senses: but every object which we can know as real must be either the product of the intellect from within, or be brought to the senses from without; therefore, space is not a real object.

57. The mastodon as a living creature has never been perceived by any of the senses of man, nor yet can it be the mere product of thought; we conclude from this that, so far as human knowledge can go, the mastodon never existed.

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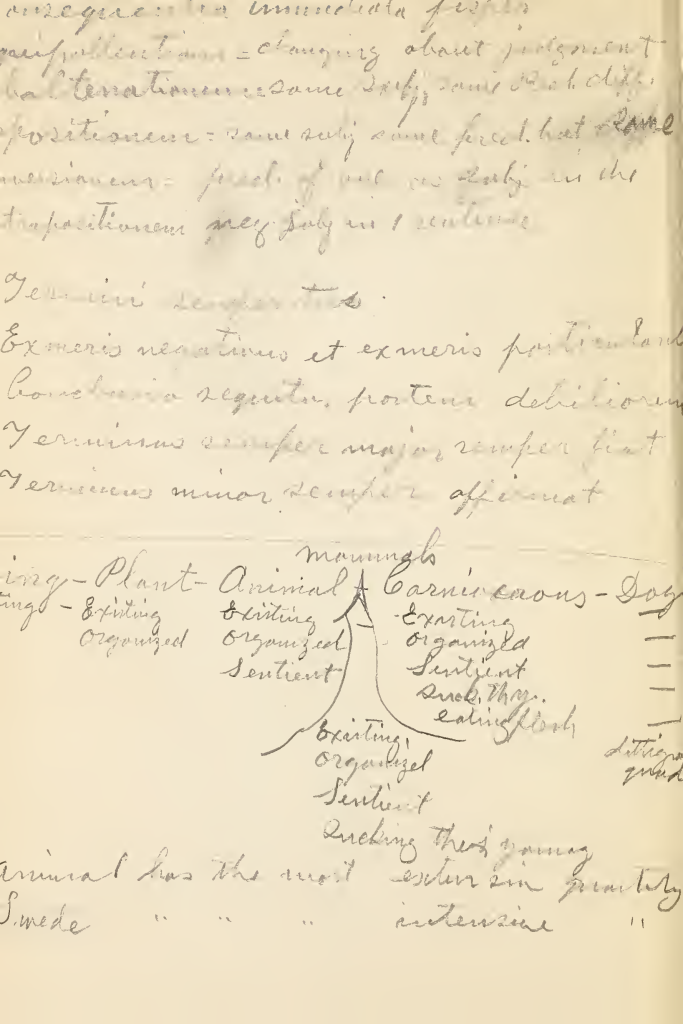
- Judgement
  - Quality
    - affirm.
    - Negative
  - Quantity
  - Relation or (Logical Form)
    - Categorical = that we can predicate something of the subject. I Law
    - Hypothetical = Pred. is a necessary consequence of the subject. II Law
    - Disjunctive = such that the pred. limits certain modifications of subj. III (This man is either black or white or yellow)
  - Modality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